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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

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Devoted to the interests of mathematics in Elementary and Secondary Schools
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Edited by William David Reeve

Operations in the Systems of Positive and Negative Numbers and Zero

By Howard F. Fehr
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New York 7, N. Y.

Despite the ready accessibility of elementary proofs[1] of the impossibility of the Euclidean trisection of an angle, every year produces its crop of would-be solvers of the problem. Fortunately, in professional circles, the proof of the impossibility of angle trisection by Euclidean methods is now so well known, that the literature need not content itself with further dissemination of the nature of the problem. The same, however, cannot be said about the knowledge of the logical development of the number systems of elementary algebra. Such developments are usually first presented in the graduate school in courses in Modern Algebra and Theory of Functions. Since many high school teachers of algebra do not take graduate work in pure mathematics, they are unaware of the logical development of algebra, and are therefore more apt to commit logical fallacies in teaching elementary algebra. Three recent articles[2] on the logical approach to $(-a)(-b)$ have indicated a lack of understanding of these logical developments as well as a confusion of ideas.

There is available in the mathematical literature a rigorous logical development of the number systems.[3] Since these treatises are not readily available to most high school teachers, and since they also use a symbolism that is alien to high school algebra and would therefore necessitate a great deal of study, there appears to be a need for a relatively simple exposition of the logical nature of algebra. A description of this sort would have to treat to some extent a logical development of some phase of number and the bearing of such a development on the teaching of elementary algebra. This article is an attempt in that direction.

In the logical development of high school plane geometry there are a set of undefined terms, a set of accepted relations between these terms which are called postulates, and a set of defined terms. The same pattern exists in the logical development of number. Beyond these basic elements are the theorems which are deduced by the use of the basic elements and previously proven theorems. For the purpose of this paper we shall assume that the natural numbers,

$$1, 2, 3, \cdots, n, \cdots$$

have been developed into a logical system through the process just described. Thus the natural numbers have been *defined;*

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they have been ordered by *definition* through the use of the symbols $=$, $<$, and $>$. Thus we know that $4=4$, that $6<8$, and that $n+1>n$ by applying the definition of order. For these natural numbers the equality relationship has been *proved* to be reflexive, symmetric, and transitive. That is, if a, b, and c are given natural numbers we have deduced that

$a=a$ (reflexivity)

If $a=b$, then $b=a$ (symmetry)

If $a=b$, and $b=c$, then $a=c$ (transitivity)

The reflexive property has an analogue in plane geometry in the identity element when we say, "line AB equals line AB, by identity."

For the natural numbers we have *defined* the operation addition. From this definition we have derived the addition facts learned in our early school years. We have also *defined* the operation multiplication. This definition enables us to derive the multiplication tables. We then *proved* that under these definitions the natural numbers obey five basic laws. They are:

I. Addition is *commutative*. $a+b=b+a$

II. Addition is *associative*. $a+(b+c)=(a+b)+c$

III. Multiplication is *commutative*. $a\cdot b=b\cdot a$

IV. Multiplication is *associative*. $a(bc)=(ab)c$

V. Multiplication is *distributive* with respect to addition. $a(b+c)=ab+ac$.

Statements I and III tell us that the order in which we add two numbers or multiply two numbers does not affect the result. Thus 2 added to 6 gives the same result as adding 6 to 2. Multiplying 3 by 4 gives the result as multiplying 4 by 3. Statement II tells us that when three numbers occur in a problem, we can choose any two numbers (in the order given) for the first addition and then add the result to the remaining number, always obtaining the same final answer. Statement IV says the same thing for multiplication. Using Statements I and II we can deduce the fact that the addition of many numbers is independent of the order in which they are combined. Thus

$$2+3+6+5=2+6+5+3$$
$$=3+5+6+2=\cdots.$$

Statements III and IV enable us to deduce the same result for multiplication. Statement V is the fundamental law of products and factoring and combines the two operations of addition and multiplication. It is important to note that these five statements are theorems; they are not postulates. Having established these five fundamental theorems, we are in a position to prove many other theorems that are universally true for all natural numbers. The totality of all these theorems is the algebra of the natural numbers.

In the foregoing the words prove and define have been italicized because logically it is important to know which statements are accepted and which are deduced as theorems. There are two other defined operations that are not always possible in the system of natural numbers. They are subtraction and division. It is the desire to make these operations always possible that leads to the creation of new and more extended number systems. In this article we shall treat the extention made through the operation of subtraction.

To subtract a number m from a number n is to find a number x which added to m will give a sum equal to n. In symbols, $n-m=x$, provided $m+x=n$ if there is such an x. For the natural numbers, $6-2$ is 4 since $2+4=6$, but $6-8$ is not possible since $8+x>8$, $8>6$, and hence $8+x>6$ for any x. If subtraction is to be an omnipossible operation we must have an entirely *new number system*. This system is developed in terms of the natural numbers but not by adding a new set of symbols to it. The new numbers will be entirely new creatures; *every one of them distinct and different from the natural numbers*. We

develop the new system under the following scheme:

1. We define the new numbers.
2. By definition, we order the new numbers. This means that we have a method of telling for two given number, whether the one number is equal to $(=)$, less than $(<)$, or greater than $(>)$ the other number.
3. The equality relationship is proved to be reflexive, symmetric, and transitive.
4. We define addition for the new numbers.
5. Using the definition of subtraction given above, we prove that subtraction is always possible for the new numbers.
6. We define multiplication for these new numbers. The so-called law of signs comes from the definition. It is *not proved.*

The development of these new numbers will then come about by using concepts defined particularly for them. We will never assume that theorems proved for the natural numbers will also hold for the new numbers. The theorems if they do hold, will be proved anew. We shall call the new numbers integers to distinguish them from natural numbers.

DEFINITION 1. *An ordered pair of natural numbers is an integer.* Thus, (6, 4), (3, 3), (4, 6) where the order in which the natural numbers are written is important, are integers. In general, (a, b) where a and b are natural numbers, is an integer. This may seem a queer way to write a number called an integer, but it is no more queer than writing $\frac{1}{8}$ or .125 for a number called a fraction. Soon we shall find a more familiar way of writing these integers.

DEFINITION 2. *The integers are ordered by the relation*

$$(a, b) \gtreqless (c, d) \text{ according as } a+d \gtreqless b+c$$

where the signs are taken in the order given. Thus, $(6, 4)=(8, 6)$ because $6+6=4+8$. Similarly, $(6, 5)>(4, 4)$ because $6+4>5+4$, and $(5, 6)<(4, 4)$ since $5+4<6+4$. Note that the new relations of order for the integers are defined in terms of addition of natural numbers which is already known.

These two definitions make it possible to deduce a number of theorems about integers from which we can obtain a more familiar symbolic representation. Thus the number (a, a) always equals the number (b, b) where a and b are any natural numbers, since $a+b=a+b$. We call this number zero and write it symbolically as **0**. It is easy to prove that $(a+1, a)>(a, a)$, and that $(a+1, a)=(b+1, b)$. Every integer in which the first paired natural number is one greater than the second natural number we write in the form $^+1$ and call it 'positive one.' In like manner we write $(a+2, a)$ in the form $^+2$, calling it 'positive two,' and so on. We call all these numbers the positive integers and they are ordered in the form $^+1, ^+2, ^+3, \cdots, ^+n, \cdots$

By definition, $(a, a+1)<(a, a)$ because $a+a<a+a+1$. We call this number 'negative one' and write it symbolically $^-1$. Similarly $(a, a+2)<(a, a+1)$ and we call the integer in which the first paired natural number is two less than the second natural number the integer 'negative two,' written $^-2$. Continuing this way we obtain $^-3$, $^-4$, etc. and call these numbers negative integers. Thus we have the positive and negative integers and zero ordered in the array

$$^-n, \cdots, ^-3, ^-2, ^-1, 0,$$
$$^+1, ^+2, ^+3, \cdots, ^+n, \cdots$$

which constitutes an entirely new system of numbers. $^+3$ *is not the natural number 3, but it is any of the infinite ordered pairs of natural numbers* $(a+3, a)$. We are now ready to operate with these new numbers which we have called integers.

DEFINITION 3. *The sum of two integers* (a, b) *and* (c, d) *is the integer* $(a+c, b+d)$. We use the same symbol $+$ to denote the operation of addition for natural numbers

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and for the integers, even though the operation is defined differently for each of these systems. The definition tells us that $(4, 1)+(10, 5)=(4+10, 1+5)$ or $(14, 6)$. Writing in the more familiar symbolism, the same problem is $^{+}3+^{+}5=^{+}8$. Similarly the definition tells us that $(4, 1)+(5, 10)=(4+5, 1+10)$ or $^{+}3+^{-}5=^{-}2$, and continuing in this way with general number pairs it is easy to establish the law of signs for addition of positive and negative integers. Again note that we *did not prove* these laws; they were implicit in the definition of addition. We are also careful to distinguish between the symbol $^{+}$ in $^{+}3$ which is a number symbol, and the sign $+$, which is a symbol of operation. We say 'positive two' for $^{+}2$, never 'plus two.' We say 'negative three' for $^{-}3$, never 'minus three.'

It can now be proved that subtraction of integers is always possible. That is given any two integers, it is always possible to find a third integer or zero which, added to the one integer, will produce the second integer. By the definition of subtraction we have $(a, b)-(c, d)=(x, y)$ if $(c, d)+(x, y)=(a, b)$. By definition 3, we have

$$(c, d)+(x, y)=(c+x, d+y).$$

By definition 2

$$(c+x, d+y)=(a, b) \text{ if } c+x+b=d+y+a.$$

The last will be a true statement if $x=a+d$ and $y=c+b$, as is easily verified by substitution. For the given integers it is always possible to find the sums $a+d$, and $c+b$, and subtraction becomes omnipossible under the formula

$$(a, b)-(c, d)=(a+d, b+c).$$

Thus $(6, 4)-(8, 2)=(6+2, 4+8)$ or $^{+}2-^{+}6=^{-}4$. By many similar examples it is easy to establish the usual rules for the subtraction of positive and negative integers. In the domain of natural numbers the problem $3-8$ is impossible; in the integers the problem $^{+}3-^{+}8$ is possible and has the answer $^{-}5$. The new system of numbers gives far more freedom of operation.

For the natural numbers $3\times5=15$. For the integers $^{+}3\times^{+}5$ is a product of two entirely different numbers for which so far no method of operating has been given. To assume that the answer is $^{+}15$ because of a corresponding product in the natural numbers is to eliminate the necessity for a definition of a new concept and proof of many theorems, which means the abandonment of logical structure. Positive 3, written as $^{+}3$, is not the natural number 3, and therefore does not mean 'three times' as used with natural numbers unless it is so defined. Before we can proceed to the operation of multiplication we must *define* the operation as follows:

DEFINITION 4. *The product of the integers* (a, b) *and* (c, d) *is the integer* $(ac+bd, ad+bc)$. Thus $(6, 3)\ (7, 2)=(42+6, 12+21)$ or $^{+}3\cdot^{+}5=^{+}15$. Similarly we have $(2, 5)\ (3, 5)=(6+25, 10+15)$ or $^{-}3\cdot^{-}2=^{+}6$. The product of these two negative numbers is a positive integer by *definition of multiplication* not by proof. Using general ordered pairs of natural numbers for the integers we can easily establish the law of signs for the product of positive and negative integers and zero.

Thus $(a, a+c)$ and $(b, b+d)$ are any two negative integers. From the definition, the product of these two numbers is the integer $(ab+ab+ad+bc+cd, ab+cb+ab+ad)$ which is greater than zero, since the first sum in the parentheses exceeds the second sum by cd. Hence, by definition, the product of two negative integers is a positive integer. The importance of this paragraph lies in the fact that *any logical approach to* $(^{-}a)\ (^{-}b)$ *must be by definition, not by proof.* With the four definitions given for the integers it is now an easy example to prove, as theorems, that the five fundamental laws of commutativity, associativity, and distribution are also true for the integers. The following excerpts give added weight to the assertions of this paragraph.

"It took a long time for mathematicians to realize that the 'rule of signs,' together with the other definitions governing negative integers and fractions cannot be 'proved.' They are created by us in order to attain freedom of operation while preserving the fundamental laws of arithmetic. What can-and-must-be proved is only that on the basis of these definitions the commutative, associative and distributive laws of arithmetic are preserved."[4]

Gauss wrote to Bessel in the year 1811: "One should never forget that the functions, as well as all mathematical concepts are our own creations and that where the original definition ceases to make sense, one should not ask, what is?, but what is more convenient to assume?, so that there will always be consistency. So, for example the product of negative times negative." And Hankel, in his work already mentioned, concerning the rule of signs: "It cannot be stressed too much in the face of the generally prevailing opinion. that all these equations in formal arithmetic never can be proven; these are arbitrary conventions in favor of the preservation of formalism in calculation.

We have shown that, for the integers, it is always possible to perform the operations of addition, subtraction and multiplication. However, it is not always possible to divide two integers. The quotient of a number a by a number b is a number x such that the product bx is equal to a. Thus $^{-}8 \div {}^{+}2$ is $^{-}4$ because $^{+}2 \cdot {}^{-}4 = {}^{-}8$. However there is no answer to $^{-}8 \div {}^{-}3$ in the system of integers. To make division an omnipossible operation we must create an entirely new system of numbers, called the rational numbers. We shall not do that here but it can be found in the references appended to this article.

To attempt any logical presentation, similar to the one here given, in the high school is highly impractical and out of order. That was not the purpose of this article. High school algebra can and should be made meaningful by other methods than the use of rigorous logic. There are many ways to teach positive and negative integers that give correct concepts, are logically correct, and yet do not resort to logical rigor. How to do this would require several subsequent articles and is discussed in reference 3-a.

It is sufficient to say here, that any high school presentation should (1) not entail developments that are logically fallacious, and (2) be intuitively convincing and acceptable to the pupils. This article was presented with the hope of clearing up some logical fallacies concerning the law of signs, and to give, for those teachers who have never had the opportunity of studying it, an introduction to the nature of the logical development of number.

In the development as here presented the following points are worthy of emphasis:

1. Multiplication was defined uniquely for all the integers. We did not give one definition for positive integers (as a continued addition as given by Sites[2b]) and as a continued subtraction for a negative multiplier, and then something altogether different for the product of two negative integers. Such an approach, found in some of the elementary texts, is unnecessary to the understanding of multiplication, it is confusing, and most of all illogical.

2. We did not use a hybrid system of natural numbers and integers. We operate with the one system or with the other, not both. We did not confuse the natural number 2 with the positive integer $^{+}2$ or its general representation $(a+2, a)$. We certainly would not write an expression $2[^{+}4+(-3)]=2$. All the numbers are integers, and then they should all be signed, or they are natural numbers, and then there should be only signs of operation. If we do write $^{+}2[^{+}4+{}^{-}3]$ and reduce this to $^{+}2 \cdot {}^{+}1$ we still need a definition of these two integers before we can find the product. Thus the postulational approach by Van Engen[2a] is illogical in the use of a hybrid system. It is also illogical in its postulational approach.

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3. We do not postulate the associative, commutative, and distributive laws. In fact, as stated before, we prove these laws for the integers, and they are proved anew for each newly created number system. To assume that the laws do hold for all new number systems is putting the cart before the horse. Indeed, under a different definition of multiplication, we can develop a logically consistent system of numbers in which the commutative law of multiplication no longer holds. Such systems have been developed and prove useful. Postulating these five laws for all number systems (or that the laws proved for positive integral exponents must therefore hold for all other types of exponents) is logically unsound.

4. We did not confuse the sign $-$ for negative integers with the sign $-$ for the operation of subtraction. Thus we do not say that: $(20-7)\ (20-3)=400-200$? 21, where the question mark indicates we must put in the proper sign to make the result true; and since this must be $+21$ consistency demands that $(-7)\ (-3)$ must be $+21$.[2-6] The numbers occurring in the first product are natural numbers and the two signs are those of subtraction. From this we cannot suddenly change to integers and addition if we are consistent in our development. In fact, the product $(a-b)\ (c-d)$, where a, b, c, and d are natural numbers is possible if and only if $a>b$ and $c>d$, and under this condition $(a-b)\ (c-d)=(ac+bd)-(bc+ad)$ where the letters represent natural numbers and the signs are those of operations. Further, if we have a product of integers $(^{+}6-^{+}3)$ $(^{+}5-^{+}2)$ we can show that this is equivalent to the product $(^{+}6+^{-}3)\ (^{+}5+^{-}2)$ but to evaluate this product we must first have a definition for the multiplication of two negative integers, two positive integers, and a positive and a negative integer. Multiplication is defined anew for each newly created number system, it is not proved.

5. In the training of our teachers of high school mathematics there appears to be a real need for a careful study of the logic of algebra as well as a study of the logic of geometry.

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By H. C. Christofferson
Miami University, Oxford, Ohio

Installment buying is again beginning to go on a rampage as it did in the 1920's. A recent news release estimates an increase of $170,000,000 per month. A glance at a daily newspaper or a few hours spent in shops where radio phonographs, automatic washers and cars are sold, will reveal the amazing and alarming trend.

If a taxi driver wrecks his taxi or a laundress wrecks her washing machine, and neither has any savings or capital with which to buy a new one, installment payments may make it possible to earn a living. Under these conditions, a high interest rate concealed in a carrying charge may be justified. However, for luxury items that people may want but not need, the program is difficult to defend for a large majority of people in contrast to a pay-as-you-go policy. Not only are large interest rates charged under cover of insurance and carrying charges, but, at this time of rising prices, an inflated demand created through this dangerously easy credit will tend to push prices higher and at an accelerated rate. To many the inevitable result will be: selling boom—crash—depression.

All teachers, but especially teachers of social studies, business arithmetic and mathematics, can now make a significant contribution to our society by emphasizing not only the high rates of interest, but also the dangers to our society which are inherent in this artificial stimulation of demand. The formulas which are available for determination of the interest rate are so involved and require so much computation that they are of little use to a purchaser who is wavering between sensible buying and buying items he can ill afford. A short formula for use in rapid calculation is badly needed. The following analysis reveals three ways of computing interest rates and concludes with an approximation formula that is adequate for common use to reveal the rate of interest which lies concealed in the carrying charge.

The following specific situation will provide a basis for analysis and generalization and will supply the formulas which are necessary to reveal the relationships needed. A washer lists at $90 cash but shows a 6% carrying charge for deferred payments, $5.40, making the installment price $95.40. This is paid by a cash down payment of $14.40 and monthly payments of $13.50 per month for 6 months.

Varied rates of interest are found by varying the time when the interest is paid. The book balance is $81. If this is considered principal, then the interest is paid in the down payment. If the down payment of $14.40 is credited on the $90, then the principal balance is $75.60. This is the principal amount which one would need to borrow to pay cash. With this interpretation, the interest is still owed and can be paid in one of two ways, either monthly or in the last payment. There are three ways to pay the carrying charge: (1) pay interest first out of the down payment; (2) distribute the charge, paying it monthly or weekly; (3) pay interest last, out of the last payment. These three analyses are given in the table on page 178.

Note in the columns marked "Interest Distributed" that $.90 interest is paid each month and that the balance of the $13.50 monthly payment, $12.60, is applied on

INSTALLMENT BUYING PROBLEM FOR ANALYSIS

Washer \$90—cash or deferred payments charge 6% or \$5.40
Down payment—\$14.40 monthly payments, \$13.50 for 6 mo.

Time	Monthly Payments	(1) Interest First	(2) Interest Distributed		(3) Interest Last
		Book Balance	Prin. Bal.	Int. Bal.	Prin. Balance
1st Mo.	13.50	81.00	75.60	5.40	75.60
2nd Mo.	13.50	67.50	63.00	4.50	62.10
3rd Mo.	13.50	54.00	50.40	3.60	48.60
4th Mo.	13.50	40.50	37.80	2.70	35.10
5th Mo.	13.50	27.00	25.20	1.80	21.60
6th Mo.	13.50	13.50	12.60	.90	8.10
	81.00	$\frac{81.00+13.50}{2}$	$\frac{75.60+12.60}{2}\rightarrow$		$\frac{75.60+8.10}{2}$
Average Principal $(B+L)/2$		47.25	44.10		41.85

the principal. In the last column, "Interest Last," the \$5.40 interest is not paid until the end. The monthly payments of \$13.50 are all applied on the \$75.60 leaving for the sixth month only \$8.10 in principal to be paid. The \$13.50 payment pays this \$8.10 balance in principal and the \$5.40 in interest.

Probably the simplest way to compute the interest rate is in terms of average principal debt. Money is owed for 6 months, but in a changing amount, the principal may be considered as the average of these amounts. One way to get the average is to add the balances and divide by the number of payments. A shorter way, for numbers in simple series, is to add the first and last balances and divide by 2, $(B+L)/2$.

INTEREST RATE COMPUTATION

Let p = average principal, i = \$5.40, t = 0.5 year.

R_1 = rate, interest paid first, R_2 = rate with interest distributed, and R_3 = rate with interest paid last.

(1) $i = prt$ or $r = \dfrac{i}{pt}$

(2) $R_1 = \dfrac{5.40}{47.25 \times 0.5} = \dfrac{10.80}{47.25} = 22.8\%$

(3) $R_2 = \dfrac{5.40}{44.10 \times 0.5} = \dfrac{10.80}{44.10} = 24.5\%$

(4) $R_3 = \dfrac{5.40}{41.85 \times 0.5} = \dfrac{10.80}{41.85} = 25.8\%$.

General Formula

Let B = first balance, L = last balance, i = carrying charge.

B has two interpretations and L has three, dependent upon the considerations given previously. See the table preceding:

$B = 81.00$ or 75.60,

$L = 13.50$, 12.60, or 8.10.

$$R = \frac{i}{\frac{B+L \cdot t}{2}} = \frac{2i}{(B+L)t}.$$

APPROXIMATIONS FORMULA

In the above problem, the important consideration is not whether the real rate is 22.8%, 24.5% or 25.8%, but the evident fact that it is about 4 times the apparent 6% rate. This reveals the need for a rapid-fire approximate rate. Since the down payment is usually at least as much as a monthly payment, $B+L$ will approximately equal the cash price, or, the average principal is approximately half the cash price, \$45.00 against 47.25, 44.10 and 41.85. Suppose we use P for cash price, r for apparent rate, R for real or true rate of interest involved in the carrying charge.

(1) $R = \dfrac{2i}{(B+L)t}$ General formula just derived.

(2) $B+L=P,\ i=Pr$

(3) $R=\dfrac{2Pr}{Pt}$ (approximately). Substitute (2) in (1)

(4) $R=\dfrac{2r}{t}$ final approximation formula, which may also be written

(5) $R=2r\cdot\dfrac{1}{t}$.

Application:

$$R=\frac{2(6\%)}{0.5}=24\%, \text{ or } 2\times 6\%\times\frac{1}{0.5}=24\%.$$

Rule: *The true rate of interest in any installment proposition is approximately twice the apparent rate divided by the time in years,* or, *twice the apparent rate times the reciprocal of the time stated as a fraction of a year.*

Further application:

Suppose the apparent rate 10% and 4 monthly payments, $\frac{1}{3}$ year. Then,

$$R=\frac{2(10\%)}{\frac{1}{3}}=60\%.$$

Or, suppose the charge is 10%, but the time 12 months. Then,

$$R=\frac{2(10\%)}{1}=20\%.$$

This formula makes sense if one just does a bit of common sense thinking. Since the average principal is about half the cash price and the apparent rate is figured on the cash price or first balance, that alone will double the rate. Then, too, the apparent rate is figured on the yearly basis. If the real basis is less than a year, that again would affect the real rate, and inversely with the time. If the real time is a half year, the apparent rate would double again. Hence, the real 24% from the apparent 6%.

If the rate for the carrying charge is figured on the unpaid principal, as it often is, instead of on the cash price, a slightly more accurate formula can be approximated. That would mean 6% of the $75.60 in the given problem or $4.54 as a carrying charge. The formula will be based on the assumption that the interest is paid first or distributed, not paid last. Use n = number of payments.

(1) $R=\dfrac{2i}{(B+L)t}$ · General formula.

(2) $B=nL,\ i=rB=rnL$

(3) $R=\dfrac{2rnL}{(nL+L)t}=\dfrac{2nrL}{L(n+1)t}$.

Substituting (2) in (1) and factoring.

(4) $R=\dfrac{2nr}{(n+1)t}=\dfrac{2n}{n+1}\cdot\dfrac{r}{t}$. Simplifying (3).

Note that $2n/n+1$ in this formula is 2 in the other formula. For the conditions mentioned, this gives a closer approximation.

If r, the apparent rate, is not given, it can readily be obtained by dividing the carrying charge, i, by the unpaid balance, B or P. The formula for approximation then becomes the following:

(1) $R=\dfrac{2r}{t}=\dfrac{2}{t}\times r$

(2) $r=\dfrac{i}{B}$

(3) $R=\dfrac{2}{t}\times\dfrac{i}{B}$

(4) Or, $R=\dfrac{2i}{Bt}$.

Application to Sample Cases

1. A garage in a northern state had a table showing deferred payment plans for 3 months to 12 months and for original bills of $50 to $500. A $50 bill would be paid in 3 months by $18.71 per month, making a total of $56.13 or $6.13 charge for carrying the account for 3 months. $6.13 is 12.26% of $50, and on the surface, this seems a small interest rate for a man who does have a car but does not have $50 for a repair bill.

$$R=\frac{2}{t}\cdot\frac{i}{B}=\frac{2}{\frac{1}{4}}\times\frac{6.13}{50}=8\times 12.26\%$$

$=98.08\%$, the real interest charge involved.

2. For a \$500 bill paid in 12 months the monthly payment at this garage would be \$44.59 or a total of \$535.08, a carrying charge of \$35.08 apparently 7.0% of the \$500 debt.

$$R=\frac{2}{t}\times\frac{35.08}{500}=\frac{2}{1}\times 7.0\%=14.0\%.$$

This is a reasonable rate for such a large bill and reveals that this particular garage penalizes the man who cannot pay cash for a small bill.

3. A radio phonograph is marked \$325.00 terms \$75 cash 6% on balance for deferred payments in 5 months.

Carrying charge $=6\%$ of $\$250=15$.
Book balance $=\$250+15=\$265$
Monthly payments $=265/5=\$53$.

$$R=\frac{2i}{t}=\frac{2\times 6\%}{5/12}=2\times 6\%\times\frac{12}{5}$$

$$=\frac{144\%}{5}=28.8\%.$$

Or,

$$R=\frac{2n}{n+1}\cdot\frac{r}{t}=\frac{10}{6}\cdot\frac{6\%}{5/12}$$

$$=\frac{10}{6}\times 6\%\times\frac{12}{5}=24\%$$

Or, accurately,

$$R=\frac{2i}{(B+L)t}=\frac{2\times 15}{(250+50)5/12}=24\%$$

$L=\$50$ if interest is distributed at \$3.00 per month.
$L=\$38$ if interest is paid at the end. Then,

$$R=\frac{2\times 15}{(250+38)5/12}=25\%.$$

The short formula exaggerates a bit if interest is computed on the unpaid principal balance. In this case, the second short formula could readily be used. Often the carrying charge is figured as a per cent of the total price, as for example, a discount for cash. On the average, the short formula will be in error by less than $\frac{1}{12}$ of the computed rate. For example, the approximate rate of 24% from the table data differs from 25.8%, 24.5% and 22.8% by less than $\frac{1}{12}$ of 24%. However, for a quick estimate of actual rate of interest in installment offering, $R=2r/t$ or $R=2/t\times i/B$ provides an easy way of discovering the presence of exorbitant rates and should serve as a deterrent for hasty buyers.

Indiana University Workshop in Mathematics

INDIANA UNIVERSITY will hold its second annual Workshop for Mathematics Teachers June 20 to July 2. Graduate credit will be given for those who desire it. There will be exhibits, lectures, group discussions, analysis of problems brought in by the members. Programs will be available about May 1. If you wish a program, please write Philip Peak, University School, Bloomington, Indiana.

Aptitude Tests in Relation to the Teaching of Plane Geometry

By ESTHER F. GIBNEY
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THE classroom teacher who receives her greatest reward in the satisfaction of knowing she has done her utmost to obtain desired results in the instruction of her pupils is continuously examining her methods and techniques of instruction with an eye to their improvement. If results are not of sufficiently high quality, she seeks all possible reasons for the lack of pupil achievement and attempts to adapt her procedures to the situation.

For some time the members of the mathematics department of Roosevelt High School, Chicago, were dissatisfied with the results of their teaching of plane geometry. The pupil population of Roosevelt is composed of students who hope and plan to go to college, and a large proportion of them do go shortly after their high school graduation. Since colleges in general require a year each of algebra and geometry for entrance, a large number of Roosevelt students elect these two years of mathematics. It is then the problem of the classroom teacher to give them a satisfactory experience in those stages of mathematics.

The dissatisfaction with pupil achievement in geometry was the topic of discussion of the department for considerable time. Individual teachers tested various teaching devices. Emphasis on reading and its relation to mathematics was made by all. The fundamentals of arithmetic were systematically reviewed. In the spring of 1947, a study of the marks given by the teachers of six classes of beginning geometry was made and related to the test data on file in the adjustment office. The data, when tabulated and examined, were significant and suggested a course of procedure and a policy for programming that were later followed.

The distribution of marks for the geometry classes for the second semester of the school year 1946–1947 follows in Table I.

TABLE I

Distribution of Marks for Six Geometry 1 Classes at Roosevelt High School for the Semester ending June, 1947

School Mark	Number of Pupils Receiving Mark
S (95–100)	14
E (88–94)	34
G (81–87)	50
F (75–80)	86
D (Failure)	17
Dropped	6
Left	5
Total	212

An examination of the above table shows that while ninety-eight pupils received average or better than average marks, one hundred three received a mere passing rating or failed. Six had dropped the subject and five had left school.

There arose the question, then, as to what were the reasons for such low ratings for more than half the pupils. Since classes in geometry were composed of pupils of all levels of intelligence on a heterogeneous basis, the factor of ability and aptitude could not be disregarded. Therefore the test data available in the Adjustment office was consulted.

Data were available for all pupils, though not in the same form nor from the same standard tests. The most frequently appearing data was from the Chicago Primary Abilities Tests, which had been used in earlier testing of seventy-two of the pupils who had received marks of F, or merely passing.

The Chicago Primary Abilities Tests measure the factor quotients in the areas

of number, spatial relationships, reasoning, verbal understanding, work fluency, and memory. Although all of these factors function in learning, yet certain ones are more frequently employed in, and more pertinent to the learning of mathematics. The factor quotients were tabulated for the seventy-two students and studied for their implications. They follow in Table II.

TABLE II

Factor Quotients of Seventy-two Pupils receiving a Mark of F in Geometry 1 as Determined by the Chicago Tests of Primary Abilities

Factor Quotients	Number	Special Relationship	Reasoning	Verbal Understanding	Word Fluency	Memory	Total
160–169	0	0	1	0	0	1	2
150–159	7	5	9	4	3	11	39
140–149	11	3	9	4	6	6	39
130–139	10	3	10	4	8	2	37
120–129	10	6	8	10	10	3	57
110–119	11	9	7	13	12	11	63
100–109	6	9	8	15	11	7	47
90–99	8	6	9	12	7	4	46
80–89	5	13	2	8	9	2	39
70–79	4	8	6	1	2	7	28
60–69	0	3	0	0	3	7	13
50–59	0	2	1	0	1	1	5
40–49	0	0	0	1	0	5	6
30–39	0	0	1	0	0	0	1
20–29	0	0	0	0	0	0	0
10–19	0	0	0	0	0	0	0
0–9	0	1	0	0	0	1	2
*******	***	***	***	***	***	***	***
–42	0	0	0	0	0	1	1
–46	0	1	0	0	0	1	2
***	***	***	***	***	***	***	***
–67	0	1	1	0	0	1	2
–69	0	1	1	0	0	1	3
***	***	***	***	***	***	***	***
–78	0	1	0	0	0	0	1
Total	72	72	72	72	72	72	432

From the data of Table II, nine of the pupils receiving a mere passing mark had a factor quotient below ninety in Number, thirty-one below ninety in Spatial Relationship, eleven below ninety in Reasoning, ten in Verbal Understanding, fifteen in Word Fluency, and twenty-seven in Memory. Of course, all of these low scores did not belong to the same individual pupils. While a low factor quotient in Spatial Relationship occurred most frequently, yet a low factor quotient in Memory was a close second.

As a result of this survey of marks and factor quotients, the members of the Mathematics Department decided to attempt to improve the achievement of pupils with low geometric ability. Since no one wished to risk classification of pupils on the basis of one factor quotient and obtaining the data from individual files in the adjustment office involved the expenditure of too great an amount of time for classroom teachers with a full teaching load, the field of tests was explored for one test that could be easily administered and scored and that would contain elements of all possible factors of intelligence. Accordingly, through the cooperation of the Adjustment Department, the Lee Test of Geometric Aptitude was obtained and administered to the students in the classes of Algebra 2, in November 1947.[1]

The test manual states, "To provide a basis for determining whether or not a student should take geometry, a critical

[1] Lee Test of Geometric Aptitude, Dorris M. Lee and J. Murray Lee, published by California Test Bureau, Los Angeles, Calif

low score of 24 has been set. Students getting below 24 should be counseled against taking geometry." While this is good advice, yet such counsel is not always acceptable to a pupil who has aspirations for a college education and whose parents are eager to provide their children with the advantages of further educational opportunities.

The results of the aptitude test were as follows:

TABLE III

Scores on the Lee Geometric Aptitude Test Given to 157 Pupils in Algebra 2 Classes at Roosevelt High School, November 1947

Score	Number of Pupils
68–71	1
64–67	5
60–63	2
56–59	5
52–55	7
48–51	3
44–47	10
40–43	18
36–39	13
32–35	18
28–31	15
24–27	11
20–23	20
16–19	10
12–15	10
8–11	6
4–7	2
0–3	1
Total	157

Of the above, there were forty-nine students with aptitude scores below the critical score of twenty-four. Two possibilities presented themselves to provide for instruction for these individuals: to accept the usual procedure of heterogeneous grouping and attempt to give attention to their needs in such a class, or to segregate those with low aptitude scores in two classes and attempt to adapt instruction to their capacities and possible rate of progress. The second alternative was accepted. Two teachers volunteered to take the special classes, and with the co-operation of the principal, these classes were scheduled for the second semester of the school year, 1947–1948.

The teaching of such a class has its peculiar advantages and its peculiar problems. The students were not informed of their scores, of course, but they were told that the test had revealed they would have difficulty with the subject. They were impressed with the fact that so much trouble had been taken to meet their difficulties and were very appreciative of the efforts made. Motivation, therefore, was not a problem, and almost without exception each member of the special classes applied himself to the learning process with enthusiasm.

Of necessity, methods of teaching had to be modified to fit the situation, often on the spur of the moment. Just what constitutes geometric aptitude is uncertain; it may be the result of training in observation of shape and form; it certainly involves ability to form abstract concepts; and it requires the ability to reason. With these abilities, it requires facility in reading and expressing mathematical thoughts. Both teachers used a modified laboratory approach to the subject. Scissors and paper were standard equipment, along with the usual straight-edge, compass, and protractor. Wherever and whenever possible, concrete illustrations were made with objects. Pupils were encouraged to make their own models.

One of the most difficult topics in geometry is the development of the concept of congruency of triangles. In this instance, pupils constructed each of the cases of congruency with compass and ruler, cut the figures out of their paper, and actually placed one upon another in the order of construction. When the idea had been made clear to all, then came the formal demonstration as an exercise in oral composition.

The concept of the triangle as the only rigid figure was actually demonstrated by the use of a model made in woodshop by a member of one class. Its use in construction of buildings, and bridges, and airplanes, was illustrated by pictures gathered from current periodicals. Paper folding to develop the idea of bisection of angles, the concurrency of lines within a

triangle, the properties of parellograms, was employed. Many of these valuable aids to teaching are to be found in the Eighteenth Yearbook of the National Council of Mathematics.[2]

It was found that these children could develop the ability to solve original problems that involved the congruency of one sets of triangles; if the proof of an original exercise required the congruency of two sets of triangles, they could think through the proof as a group with the assistance of the instructor but the effort and time consumed appalled them in retrospect. Life just should not be so complicated.

Both classes covered the course of study requirements for the first semester with no topics omitted. However, since it required more time to develop geometric concepts, the more difficult original exercises which delight the mathematician were not attempted. At all times, the aim of teaching was to develop understanding within the limits of the pupil's ability.

In order to evaluate this experiment in teaching pupils of low geometric aptitude in segregated classes, at the close of the first semester a standard test of achievement[3] was administered to all Geometry 1 classes at Roosevelt High School. The percentile rank on this achievement test for all students who had been given the aptitude test follows in Table IV.

Someone, of course, will look at the total of Table IV and compare it with that of Table III and wonder what happened to the forty-five additional pupils who took the aptitude test originally. Some of them failed in Algebra 2 and repeated that subject. Some of them elected something other than Geometry 1 and did not take the achievement test given at the end of the first semester of the subject. Some of them had moved to other high school districts. The pupil load in the six geometry classes was more than the total shown in the above table but the additional pupils were those who had previously failed in the subject, or those who had transferred to, or moved into, the Roosevelt High School district. Aptitude test scores therefore were not available for all students in the Geometry 1 classes tested.

TABLE IV

Percentile Rank on Becker-Schrammel Plane Geometry Test I of Ppuils tested for Geometric Aptitude

Percentile Rank	Number of Pupils in Special Classes	Number of Pupils in Regular Classes	Total
96–100	0	0	0
91–95	0	5	5
86–90	0	1	1
81–85	1	5	6
76–80	0	4	4
71–75	2	5	7
66–70	1	10	11
61–65	2	9	11
56–60	0	5	5
51–55	1	10	11
46–50	2	10	12
41–45	2	6	8
36–40	5	3	8
31–35	2	2	4
26–30	6	2	8
21–25	5	3	8
16–20	2	1	3
11–15	3	2	5
6–10	3	4	7
0– 5	6	2	8
Totals	43	89	132

An examination of the percentile ranks in the above table shows a rather acceptable distribution of scores. However, percentile ranks are more intelligible when translated into terms of school marks. These follow in Table V.

The success of the modified laboratory procedure as determined by the test is evident. Only five of the students in the special classes received a failure rating, while five of those with an aptitude score which would normally predict success failed.

Since a rating on a single achievement test is not the customary method of marking pupils and since attitude, daily preparation, and marks on tests and required

[2] Multi-Sensory Aids in the Teaching of Mathematics; Bureau of Publications, Teachers College, Columbia University, New York.

[3] Becker-Schrammel Plane Geometry Test, Test I, Form A; Bureau of Educational Measurements, Kansas State Teachers College, Emporia, Kan., 1934.

TABLE V

School Marks on Becker Schrammel Achievement Test in Six Geometry 1 Classes at Roosevelt High School for Semester Ending June, 1948

Semester Mark	Number Rec. Mark with Apt. Score below 24	Percent Receiving Mark	Number Receiving Mark with Apt. Score above 24	Percent Receiving Mark	Total	Per cent of Total Receiving Mark
S (95–100)	0	0	5	5.4	5	3.8
E (88–94)	2	5	12	13.1	14	10.6
G (81–87)	24	60	63	68.5	87	75.9
F (75–80)	9	22.5	7	7.6	16	12.1
D (below 75)	5	12.5	5	5.4	10	7.6
Total	40	100.00	92	100.0	132	100.0

work are all considered when a teacher makes her final evaluation of a pupil's work, the semester marks of these pupils are significant to this study. They follow in table VI:

In the segregated classes, those who would otherwise have experienced a much higher mortality reduced their failures to ten percent, three percent of the total tested for aptitude. It is questionable if they would have had as successful an experience in regular classes, competing with the remainder of the pupils. In commenting on the experiment, one of the teachers of the segregated classes wrote, "I believe they enjoyed geometry, and I can't say that for a slow child in a heterogeneous group."

In summary, while much has been written on the subject of segregation of classes on the basis of ability, yet those who participated in this experiment as teachers of the groups with an aptitude score below the critical one felt that it was worth while from the standpoint of results and of pupil satisfaction. Although data which cover the areas measured by the aptitude test are on file in the adjustment office yet the administration and scoring of the tests and classification of the data secured were accomplished with more efficiency and dispatch than would have been possible in searching through the files of the individual pupils. Methods of teaching could be more readily adapted to the ability of each group and individual difficulties could be more readily ascertained since they were more obvious to the teacher. The child who would otherwise hesitate to exhibit his lack of understanding in a heterogeneous group lost his hesitancy in a group of his own kind.

While it might not be possible to set up such an experiment in every school, yet wherever and whenever it is possible those who participate will probably find

TABLE VI

Semester Marks for Pupils with Geometric Aptitude Scores in Six Geometry 1 Classes at Roosevelt High School, for Semester Ending June, 1948

Semester Mark	Number Receiving Mark with Aptitude Score below 24	Per cent Receiving Mark	Number Receiving Mark with Aptitude Score above 24	Per cent Receiving Mark	Total	Per cent of Total Receiving Mark
S (95–100)	0	0	8	8.7	8	6.0
E (88–94)	4	10.0	28	30.4	32	24.3
G (81–87)	15	37.5	30	32.6	45	34.1
F (75–80)	17	42.5	36	28.3	43	32.6
D (Below 75)	4	10.0	0	0.0	4	3.0
Total	40	100.0	92	100.0	132	100.0

that their efforts are repaid in the satisfaction they themselves receive and in the enjoyment the students evidence in their accomplishments.

ACKNOWLEDGMENTS

Appreciation must be expressed to those who participated:

To Mr. John L. O'Brien, Principal, who made it administratively possible to set up the special classes.

To Miss Elizabeth Waters, Adjustment Teacher, who helped secure the tests and aided in their administration and scoring.

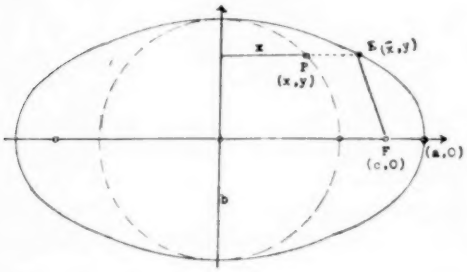
To Miss Georgia Birtman who taught the other special class.

To Miss Lettie Bristol and Mr. Wilfred Hendershot, who taught the four remaining geometry classes and who administered the achievement tests and submitted the scores.

The Ellipse from the Circle

By WILLIAM R. RANSOM, *Tufts College, Medford, Mass.*

For many purposes it is advantageous to regard the ellipse as the result of applying a one way stretch (or shrink) to the abscissas (or ordinates) of a circle. The cartesian equation of the ellipse is commonly deduced from its focal property; the algebraic work in this involves twice squaring and the appearance and disappearance of fourth degree terms. The equation is much more simple and directly obtained by considering the stretching of a circle; moreover the focal property emerges from that point of view in a simple and natural way.



If the circle $x^2+y^2=b^2$ is elongated so that the horizontal lengths are changed in the ratio of b to a, the abscissa, x, of a point on the circle is changed to $\bar{x}=ax/b$, whence $x=b\bar{x}/a$. On substituting this into the circle equation we get $(b\bar{x}/a)^2+y^2=b^2$, or $(x/a)^2+(y/b)^2=1$ as the ellipse equation.

Now we ask about the distance from a point, E, (x, y), on the ellipse to a point, F, (c, O) on its long axis. The square of this distance is $(ax/b-c)^2+y^2$, which may be transformed as follows:

$$(EF)^2=(a^2/b^2)x^2-2acx/b+c^2+b^2-x^2$$

$$=\frac{a^2-b^2}{b^2}x^2-2\frac{ac}{b}x+c^2+b^2$$

For this to be a perfect square, we must choose c so that

$$\frac{a^2-b^2}{b^2}(c^2+b^2)=(ac/b)^2$$

which gives at once, $c^2=a^2-b^2$. Using this value we get

$$(EF)^2=c^2x^2/b^2-2acx/b+a^2$$

To choose the positive root for EF, we note that $c<a$, and since P is on the circle $x<b$, whence $cx<ab$ and $cx/b<a$. Hence for the positive root we have $EF=a-cx/b$.

Similarly, if we replace F by F' at $(-c, O)$, we get $EF'=a+cx/b$. The focal property, $EF+EF'=2a$, follows immediately.

A Department of Mathematics Prepares an Exhibit

By Myron F. Rosskopf
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and
Gaylord C. Montgomery
John Burroughs School, Clayton, Mo.

Various individuals place the study of mathematics at all points on the scale from zero to infinity when its essential value in human endeavor is considered. There are those who recognize no practical uses for mathematics and there are those who believe that its study holds the secrets of life.

Teachers of mathematics have had cause for considerable concern during the past two decades because of the pointed attacks on the effectiveness and worth of their teaching. These attacks have not been without value. Study of specific problems raised by the questioning of mathematics teaching has been fruitful of results with implications in other areas.

In planning its share in a school-wide exhibit of pupil work, a mathematics department realizes that it has a twofold duty:—(1) there is a beauty to mathematical results and mathematical thinking which it is well to stress at every opportunity; (2) mathematics is practical. The study of mathematics arouses in some students an enthusiasm which is worthwhile and satisfying; these individuals who sense the internal beauty of our subject must not be slighted when work is displayed. On the other hand, there are many who can see no further than the application of mathematics; these, too, must have a chance to plead their cause.

Many mathematics teachers consider it "taking time" from their instruction in the fundamental manipulative skills when discussions arise concerning intelligent citizenship or processes of government or full employment or distribution of goods. These topics have mathematical implications and are included in our courses of study because they contribute to a pupil's growing understanding of the social and economic system in which he lives. An understanding of taxes (income, amusement, sales, and the like), interest rates in banking and installment buying or home financing must be made important to the pupil at his level. The problems must be meaningful; that is, the materials used in the classroom must be such that outside the classroom the pupil will recognize applications of the procedures taught. Otherwise these ideas are not likely to matter much to him. Actual experiences are more effective in learning than are intellectual discussions only. Classroom morale can be developed to a high point by means of pupil planning for an exhibit. With the teacher guiding the class discussions, ideas for activities in connection with the exhibit will come from the pupils themselves. Since they feel that they have had a real part in purposing their activities, they are more likely to maintain a high level of work throughout the period of preparation.

But there is more to an exhibit of a school's work that what goes on in the individual classrooms. A year before the all-important date, the faculty begins its consideration of plans. Procedures are discussed at several faculty meetings by the faculty committee in charge. Questionnaires are sent out to each faculty member, a deadline for their return having been set in advance. From the responses to these questionnaires the Planning Committee is able to allocate space for the exhibit, prepare a tentative time schedule,

and estimate the quantity of needed supplies. At the beginning the faculty is somewhat apathetic; the exhibit date is far in the future and there seems to be no immediate need for planning. But committee reports to the faculty offer so much data for consideration and examination that there soon results a quickening of interest. Departments worry about supplies, too; students who desire to construct elaborate exhibits must begin to work; it is obvious that posters, signs, and labels cannot be prepared the day before the exhibit.

As a result of these faculty-wide discussions certain individuals and certain departments are given responsibility for one phase of the planning and authority to carry out its recommendations. For example, one person is put in charge of the program and the requests for students' time; these two go together since one job cannot be accomplished without the other. All members of the faculty submit to him the names of the students desired and the time intervals they are needed (that is, half-hour periods during the scheduled time of the exhibit). It is his task to try to satisfy everyone as nearly as possible. He is responsible for seeing that every pupil has a share. This is an exhibit which represents total school participation. It is obvious there are several requests for the same pupils. All such problems are solved by amicable compromise among the people (including the pupils) concerned.

As another example of organization, consider the problem of posters, signs, and labels. Ordinarily one would say that this is in the domain of the art and industrial arts departments; but wait a minute; those departments have their own exhibits to get ready. How can they devote a large share of their time to satisfying the needs of all fields of study in the school? This is worked out to everyone's satisfaction when one of the men in the industrial arts department volunteers to be responsible for all lettering. Teachers make their requests of him, not verbally, of course, but written out in detail so that there will be no mistakes. Each teacher tells this man when the label or poster or sign has to be ready, the dimensions, and the size of lettering desired. With such specific information he is able to organize a corps of students to work according to a time schedule.

Being made up of people trained to analyze problems systematically, the mathematics department develops a pattern of attack on its share in the exhibit. Its members know, from their previous experiences with mathematical exercises, that a step by step process of solution is in order; not everything can be left until the last month. So soon as the exhibit project is unfolded in faculty meetings a departmental meeting is scheduled. At this gathering the problem, "What does the mathematics department wish to exhibit?" is discussed at great length. No definite conclusions are reached; the possibilities are explored and the thinking of the group is directed along several paths. By the time of adjournment a future meeting is agreed upon, everyone having for his individual task the outlining of his particular desire or desires for the exhibit. Clearly the interests of the group are diverse. It will be an easy matter to obtain an exhibit with a variety of material and to divide responsibility among the members of the department.

Sufficient time is allowed between department meetings to give busy teachers an opportunity to work out their ideas in some detail. Even though one realizes they are stimulated by faculty discussions, a desire to see mathematics presented to the "public" in the most favorable light, and to have their students' ideas represented in the exhibit, it still amazes one to see the fine work which has been done. Each mathematics teacher has either a sample of work to show and explain to the rest or a detailed outline of a program of exhibit material. (It should be a matter of record that two meetings were

used to present the plans to the group and to discuss the advantages and disadvantages of each plan.) Through these discussions a synthesis of individual plans was attained which unified all the efforts to present the students' work in mathematics.

Let us be your guide along the corridors of the John Burroughs School to view the exhibit (with particular reference to mathematics, of course). These materials were selected and arranged by students in the school's mathematics classes; their teachers served as counselors and advisers only. Since the work was done by pupils from grades seven through twelve, the range of content is from drill work on the fundamental operations in arithmetic to examples showing applications of elementary calculus.

Outside the door to the classroom in which the materials are displayed is a large rigid sign. It consists of a right triangle from which hangs a rectangle; below the rectangle is a circle, which in turn supports an arrow pointing through the open door. Parents are still trying to recall the names of the geometric figures in this sign as they step through the door and see that the four walls of the room are decorated with a sequence of panels. The first wall has a large caption which runs along the tops of the panels and proclaims:

STUDENTS STUDY
MATHEMATICS
at
John Burroughs School

Below, over the first panel, is the first end of the sentence: DIRECTED BY THESE TEACHERS. Succeeding panels have the captions: USING THESE BASIC TEXTS; AIDED BY THESE INSTRUMENTS; SEEKING APPLICATIONS TO OTHER FIELDS OF STUDY; FINDING RECREATION IN ITS STUDY.

Students planned, designed, and made all the panels; it is worthwhile to describe them in some detail because of their novelty. The first panel had enlarged candid photographs of the mathematics teachers, each framed in a suitable geometric figure. Coming from the lips of each was a comic strip balloon with statements like, "Arrange your work neatly!" "Is your answer reasonable?" "What are the assumptions?" "Show all necessary computations!" "Come to my office after lunch for help." These quotations were suggested by students as most typical expressions of their respective teachers. On the table below this poster was a display of several books, indicating that teachers study, too. Some of the recent *Yearbooks of the National Council of Teachers of Mathematics* appear as well as other publications such as *Mathematics in General Education, Geometry Professionalized for Teachers* (Christofferson), *Recent Developments in the Teaching of Geometry* (Shibli), *The Meaning of Meaning* (Ogden and Richards), *Logic, The Theory of Inquiry* (Dewey).

The second poster shows an arithmetic problem in an open book, a step in the solution of a simple equation, the graph of a system of two linear equations, and last a derivative and an integral. Underneath, lying open on a table, are new copies of the texts the pupils are currently using in grades seven through twelve.

Next to this panel appears a splash of color in tempora and ink. The artists show by means of cartoons students using slide rules, a transit, sextant, compass, straight edge, and other mathematical tools. Arranged on the table below this poster one finds the instruments pictured. Parents commented favorably and were much impressed by the ability of nearby students who were alert to answer questions and to explain the use of the instruments.

The fourth poster was the most unusual of the lot. The twelfth grade girl who designed and painted it was an outstanding student of both mathematics and science. For the central theme of both the poster

and the exhibit material, she chose Chladni's classical demonstrations with vibrating plates of various physical properties. The poster itself showed a thin plate covered with sand. The plate was bolted to a heavy block, and a violin bow, held by a hand, was just leaving the plate after setting it in vibration. The table below held similar suitable materials for demonstrating the same principle. There were two violin bows, a goblet partially filled with a solution of washing soda with a few drops of dark oil added, and a phonograph record mounted at its center horizontally on a large bolt cemented into a block of concrete. When the lip of the goblet was bowed, the vibrations set up produced families of equilateral hyperbolas in the dark brown oil film which covered the solution. Similarly, when white sand was spread on the phonograph record and its edges bowed, the sand arranged itself, because of the vibrations, into clearly defined families of hyperbolas; by damping the vibrations with a touch of a finger to the edge of the record, other mathematical patterns were established.

The last poster served to introduce a more dynamic part of the exhibit. At tables along the wall were groups of students who were eager to demonstrate their skill in mathematics. At one table sat two seventh grade girls and two boys; while the former worked arithmetic problems by use of gelosia multiplications, the latter did the same examples with a set of "Napier's bones." These had been constructed of thin wood slats approximately thirty inches long. An eighth grade boy emptied his bulging pockets, revealing many games and puzzles. Many of the parents carried away with them number games to try at their leisure which this boy wrote out for them. A senior boy had his personal charts and sextant on display; he talked quietly and capably about marine navigation with those people who were interested. Two seventh grade boys showed the slide rules they had made. One was a circular and the other a cylindrical rule. These boys were eager to explain the construction as well as the use of their rules to anyone who listened. Separating the groups of students were static displays of work which gave evidence of recreational activity in mathematics. Here a solid geometry class member had made a series of mechanical drawing plates illustrating the important theorems of the course; another senior had a cleverly worked out booklet on dynamic symmetry. Other painting and mathematics students had their finished work on exhibit along with a series of drawings showing their planning of the design; it was clear at a glance that many mathematical concepts were used.

Down the center of the classroom was a double row of tables on which was exhibited material of the more traditional sort. Here parents could find examples of homework assignments, tests, and reports. Two tables were taken up with models of figures from plane and solid geometry. Some were the result of paper folding; others were made of wood, string, or wire. All made obvious to parents that their sons and daughters were careful workmen and capable of painstaking attention to detail in the accomplishment of a project. Not only was the usual work in graphs on exhibit but one class had gone to great lengths to point out by means of examples some of the pitfalls in graphical representation. One observed many adults carefully studying these clippings from journals and newspapers.

Subsequent classroom use of materials was considered when cleaning out the room the next day. Of course, some of the pupils wished to keep their work and this was allowed. Each committee of students chose one member to assist in the storing of materials. The walls of the office shared by members of the mathematics department were formerly bare. They now carry colorful posters and cartoons, which create a suitable atmosphere in which to foster interest in the study of mathematics. Students who visit the office

may handle the models remaining and discuss their properties and uses with classmates. It has been observed that these models stimulate present students to prepare either better or other materials to complete and supplement the exhibits in the classrooms as well as the departmental office. There is a gratifying response to teachers' encouragement of activity of this sort.

In summary it is clear that a successful exhibit requires planning and preparation well in advance of the exhibit date. Much learning takes place in both those pupils considered excellent in mathematics and those considered poor. All plan their projects with their teachers and are allowed to participate actively in carrying them out. Responsibility was accepted by the students for their share of the work as long as they felt they had the authority to go ahead on their own and the self assurance that the exhibit was "their" exhibit.

An Answer to Brother Robert

By EDNA BROCK CRAIG, *McKinley Junior High School, Muncie, Ind.*

THE article, Algebra Teachers "Take It Easy," by Brother B. Robert in the December 1947 issue definitely requires an answer. This is mine:

Algebra is by far the easiest and most fascinating subject in the curriculum to teach and to present to the class. It is fun to develop a new idea before a class; leading them on from step to step until they think and in reality have discovered a new truth for themselves. The teacher gets an added thrill when the thinkers in the class see even beyond the point in the lesson.

Any teacher who is self-conscious and demoralized when presenting a problem to his class has a good case of class room nerves. If his class room requires "watching like a hawk," it is an unhealthy situation, and he has neither the confidence or good will of his class.

There is no subject in the field of teaching where the instructor can be more thoroughly informed. If he has such little poise and ability that a simple error in computation destroys his equilibrium, or to quote, "becomes upset over some slight error," or "is thrown out of gear," something is definitely wrong. Any teacher who sets himself up as one who is infallible certainly misses an opportunity for a sympathetic understanding of children. Many errors in their discovery have taught the child with more emphasis than if the error had not been made.

This does not mean that there is no place for boardwork in the algebra class room. A pupil who has worked a difficult problem takes great pleasure and develops ability in addressing a group and offering his solution to the class. Certainly he would be even more embarrassed than the teacher if he were called to present new material and somehow fumbled through the work. New material could be presented more masterfully by the well informed teacher and the children will have a great deal more confidence in starting their new work than if the explanation were stumbled through to some kind of a conclusion by one no better informed than themselves.

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Applications of Mathematics in Pre-Engineering Mathematics*

By JOHN W. CELL

Professor of Mathematics in College of Engineering
N. C. State College, Raleigh, N. C.

THE use of applications for mathematics like the use of slides and models can be a motivating influence for the learning process. But any use of these tools must necessarily be kept subordinate to the real objective—that of actual instruction in mathematics. This point cannot be made too forceful and what I shall have to say about the use of applications must be understood in this perspective. The teacher of high school or college mathematics makes a serious mistake whenever he tries to use an application problem that requires an hour or more of instruction in some phase of science or technology—just as he makes a mistake when he allows a full class period to be subordinated to the construction of some model.

There are numerous books available which can aid the teacher of mathematics to obtain application problems. Some of these are listed as follows:

Applied Mathematics for Technical Students, Corrington (Harper)
Introduction to Mathematical Anslysis, 2 vol., Griffin (Houghton-Mifflin)
A Source Book of Mathematical Applications, National Council Yearbook, 17th.
Engineering Preview (McMillan)
Practical Mathematics for Home Study, Palmer & Bibb (McGraw-Hill)
Mathematics for Electricians and Radiomen, Cooke (McGraw-Hill)
Industrial Algebra and Trigonometry with Geometrical Applications, Wolfe, Mueller, & Mullikin (McGraw-Hill)
Shop Mathematics, Felker (Bruce)
Mathematical Aids for Engineers, Dull (McGraw-Hill)
Engineering Problems Illustrating Mathematics, Cell (McGraw-Hill)

A second source of applications can be had by attending a Mathematics Institute such as the very excelle... one at Duke University each summer. This Institute lasts for a period of two weeks and gives the teachers an opportunity to exchange experiences and ideas, to attend lectures by excellent speakers from industry and science, to attend classes of a refresher nature, and so on.

This coming summer we at North Carolina State College, which is the technical college of the University of North Carolina, are going to try an experiment which will last for the six weeks of the regular summer school. We hope we can aid the high school teacher of mathematics and physics in this summer session by giving an *integrated* program which will have the cooperation and teaching services of the entire engineering faculty. The three courses in his program can be described very briefly as follows:

1. One class will meet for five one hour periods and one two-hour drill period each week in mathematics. The material for this course will be largely graphical in character and will range from topics in college algebra to an explanation of the meaning of calculus.

2. A class meeting for five one hour periods and one laboratory period each week in physics will not be the traditional course in college physics but will be concerned more with tracing fundamental ideas through related phases of physics. For example, the sine wave occurs in many topics in physics—these several topics will be considered from this graphical point of view. Again the inverse square law occurs repeatedly and such related topics will be grouped together for purposes of study.

3. An introduction to *what is engineering* will be given in a third class which will

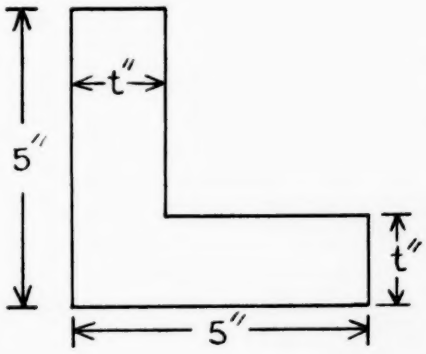
* Paper read at the Christmas meeting of the National Council of Teachers of Mathematics at Columbus, Ohio. December 30, 1948.

meet for five one hour periods and two inspection trips or laboratories each week. All of the engineering departments will cooperate in giving this course and will in turn describe the work their graduates do, the kind of mathematics their students need, and so on. Some of the laboratories will be devoted to actual engineering experiments where the teacher student will be enabled to plot the data on a graph sheet and see the fundamental law develop.

It is our considered opinion that we of the engineering colleges can help the teachers of mathematics and physics by giving such an integrated course and that these teachers will be helped in their teaching of all students and not just these going on to a study of some phase of technology.

With these general remarks out of the way I want to devote the remainder of my time to a discussion of specific topics and to make a few suggestions.

It seems to me that a great part of the elementary training in algebra is concerned with learning how to *translate* from English to algebra and back again. Herein lies one of the responsibilities of the mathematics teacher in the training of the student in English. We at N. C. State find that a great portion of our students who enter and are found to be deficient in preparation in mathematics are also deficient in the ability to read and in the ability to spell and to use the English language. Incidentally, we are now beginning to get in college some of the comic book era of students. So it seems to me that the teacher of elementary algebra should have at least one word problem every day and one on every quiz. Also the teacher should utilize every opportunity to make the students translate. Simple problems can be dictated—and don't let the students "marry" x. n is a good letter for number, t for temperature or for time, w for weight, and so on. In these same remarks let me add that education is a continuing process and each of us must keep on teaching arithmetic. I had a very serious faced college freshman ask me one evening not long since, when I happened to be walking through the hallway on the way to my office, what rule was used to obtain $\frac{2}{4}$ as the result of multiplying $\frac{1}{2}$ by 2. That sort of question is common to me as a teacher in a college and is common throughout this country of ours. Couldn't we help by making some of our problems less of the trick number type? Why do we emphasize factoring when in practice quadratic equations don't factor simply? For example: Problem: Find the thickness of the I-beam section shown in the figure if the cross-sectional area is 10 sq. in. and give your result correct to the nearest 0.01 inch.



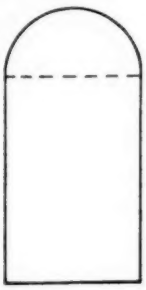
This is a good example of a several-step problem. The student must see that the I-section is composed of two rectangular sections or of a square less a square. Such a problem could equally be used in plane geometry if the principle of continued instruction is to be used.

Problem: The rectangular portion of the figure has a height twice its width and the top is semicircular. The area is 40 sq. in. Find the radius of the semicircle correct to the nearest 0.01 inch. (Figure on next page.)

This problem is simple to solve but requires a little arithmetic in the execution and it is not too early for the student to begin to have a feeling for rules of accuracy and how much accuracy to give to a particular result.

I can't help remarking that too many teachers in elementary mathematics are teaching analytic-geometry methods for sketching conics when they should be teaching point-plotting methods. The required arithmetic would have been exceedingly beneficial to some of my students.

The construction of a typical home ought to be required information of every



teacher of elementary mathematics. Uses of triangles, the carpenter's square, etc., are legion.

Applications are grand tools in teaching but anyone who has been exposed to plane geometry should never be guilty of thinking that one example constitutes a proof. Memorization of proofs, as my wife claims she did in order to pass plane geometry, should be made impossible. Again the teacher is concerned with larger motives than just plane geometry—that teacher is teaching the nature of proof. And some of this new fangled education theory is all wrong when it makes us convey to the student the idea that this new topic is going to be hard and—as a result the students always find it difficult. Abbe Dimnet in his *The Art of Thinking* (Simon-Schuster) has more education information than any college text I ever saw. He tells of his Latin teacher, who "had no idea of any directions but who possessed a tradition, said to us in perfect good faith: 'Dies and cornu being simplicity itself, you shall learn these two declensions, instead of one, for next time.' The result was that even dunces were not afraid of Latin declensions."

So many of my college freshmen have a mortal fear of algebraic fractions. I think some teacher or some system must have made them think fractions are hard. And these same students can cancel and they can cross-multiply and they can transpose. But they have difficulty adding equals to equals, or subtracting equals from equals, or multiplying both sides of an equation by the same algebraic expression. (Am I making my point clear that I firmly believe that we teachers have gone too far in short-cutting and we ought to cleave to fundamentals?)

This business of seeking applications problems for use in teaching first year high school algebra is almost like asking the first grader to read a classic—our students don't know their translation technique well enough yet to dive into applications. But the teacher can utilize formulas (an excellent collection may be found in the text on Mathematics for Radiomen and Electricians by Cooke).

Applications of the rules for radicals and exponents are difficult to find. But the student needs a philosophy of notation for their correct use. Again, a manipulation of formulas from technology may be sufficient motivation and at the same time serve an important function. Incidentally, few college freshmen can simplify $10^{\log_{10}14}$ and yet they claim to understand logarithms.

Turning to solid geometry, it seems to me that there are several objectives in such a course. One of these is the use of mensuration formulas as such. Composite body problems are easily constructed and serve as good applications. A second objective lies in reasoning with spatial objects in terms of theorems and the guy wires of a tall tower or pole are made-to-order for class instruction. A third objective should be the beginning of the ability to draw space figures on a sheet of paper and to be able to visualize the drawing into three dimensions. A fourth objective is a maturing in mathematics. Just the other day one of my analytics students came into my office and I soon

found that his difficulty lay in a complete inability to visualize. I asked him to start with two points I located in terms of (x, y, z) and to draw a rectangular box whose faces were parallel to three coordinate planes and which had the two given points as the ends of one diagonal of the box. This student found every pair of lines on his flat drawing intersected and he spent an amazingly long time getting that box drawn.

I wonder what would happen if a teacher of plane and solid geometry took analytic geometry and calculus *again* in order to see through the eyes of the teacher some of the possible uses of his fundamental courses and *to make up problems for use in that instruction.* Most every calculus text has problems which could be rephrased as in the following: *A man is walking across a bridge at 5 miles per hour while a rowboat goes underneath and at right angles at 4 miles per hour. If the man is directly over the boat at the center of the bridge and 50 feet above, how far apart are they one minute later* (assuming that both continue to move in the same directions and with the same speeds)? It seems to me that such problems could be used repeatedly in solid geometry to train the student in this art of visualizing right triangles, intersecting planes, and so on and in mixing a little algebra with geometry.

Many high schools are today teaching a course in mensuration problems in which they utilize the trigonometric functions. But teachers of such courses should carefully refrain from calling any such course a course in trigonometry unless the course includes on an equal footing the algebraic portion of trigonometry. In such a mixed course applications are readily available from science and technology. But here again the student should be learning a few formulas and learning how to manipulate them by the basic rules of algebra (if equals be added to equals, etc.). Too many teachers throw the whole book of formulas at their students—problems can, of course, be solved quicker but are we not aiming at more fundamental objectives than the solution of problems? Repeated manipulation of basic relationships to obtain others may take time but it will be time well spent. If application problems are used, the teacher should be careful that the stated problem does not require understanding in science or technology beyond the level of the student.

By the time the student of science or of technology reaches the course in college algebra his vocabulary in translating from English to algebra by way of tools from geometry and science should be sufficiently mature to allow a wider range of problems in application. Many present day texts for this course utilize such problems and others could be made available from courses in science and technology. In this course the student should be beginning to synthesize his knowledge of elementary algebra and geometry so that he can face with some assurance several-step problems where he must ascertain for himself at each step what he is to do next. The teacher at this stage should begin to avoid memorization of many new formulas and to try instead to create an understanding of processes. The all too familiar jargon that so many texts include in heavy print as the method for solution of the quadratic equation by completing the square should be avoided at all costs. For those students who are to go on to analytics where they will be required to complete the squares on x and y in $4x^2-9y^2-12x-36y=41$ and who will need to be able to complete the square on t under the radical in the integration proglem:

$$\int \frac{dt}{\sqrt{25+4t-2t^2}},$$

the rote rule is useless. For those who are not going on to these courses, I can see no value in this rote rule—if they ever need to solve a quadratic equation, the formula will be the better method.

From what I have been saying it must

be evident that I believe the best way to secure application problems would be for each teacher to have a chance, preferably during the process of earning the Master's degree, to secure background material in science, in technology, and in basic mathematics. For the teacher of high school mathematics a course which would knit freshman and sophomore college mathematics into a single unit would be far better than the traditional courses customarily given to graduate students in mathematics. And a course in mechanics could be fascinating to such a teacher if taught with the teaching aim in mind. The present day misunderstandings about graphs and the difficulties so many students have in analytics in college might very well be different if the earlier perspective were nurtured in another manner. May I close by saying that these are the objectives we have set at N. C. State for the summer course next summer?

Ninth Summer Meeting of The National Council of Teachers of Mathematics, University of Denver, Denver, Colorado August 29-September 1, 1949

The National Council of Teachers of Mathematics is holding its first summer conference since the war at the University of Denver, Denver, Colorado for four days beginning Monday, August 29, 1949. There will be sectional meetings and discussion groups devoted to topics related to the teaching of mathematics for those interested in the teaching of elementary and secondary school mathematics, junior college mathematics, and in the training of teachers of mathematics. The latest films and filmstrips in mathematics will be shown. Professor A. W. Recht of the University of Denver is local chairman for this meeting.

University Park Campus Accommodations

The meetings will take place on the University Park Campus, seven miles from the center of Denver's business section. Many Council members will prefer accommodations near the meetings. A limited number of rooms will be available in college dormitories. The charge for double and multiple rooms will be $2.00 per night per person. Some single rooms will be available at $2.50 per night. A few rooms will be available as early as Saturday night, August 27. All rooms will be available from Sunday, August 28, through Thursday night, September 1.

Meals may be obtained beginning on Monday in the University Park Campus Cafeteria at an overall club rate of $2.20 per day or at individual rates as follows: breakfast, 50 cents; lunch, 80 cents; dinner, $1.25. There will be a banquet on Wednesday evening, probably in a downtown hotel.

Reservations for rooms on the campus should be made soon. Reservations should be addressed to Professor A. W. Recht, University of Denver, Denver 10, Colorado, and should include the following information:

1. Date you plan to arrive in Denver.
2. Nights and numbers for which room reservations are desired.
3. Names of other persons with whom you plan to attend.
4. Meals you wish to take in the cafeteria.
5. Number of banquet reservations desired.

Hotels

There are many hotels and one college dormitory, Rodney Curtis Hall, available in metropolitan Denver. A list of these hotels with rates for rooms will be printed in the May issue of The Mathematics Teacher, along with directions to reach the University Park Campus. Reservations for all rooms in hotels should be made with the individual hotels.

Members of the Council, Families, and Friends

Plan now to attend this summer meeting of the National Council. Why not plan a vacation trip to cool and colorful Colorado for this summer and make these meetings the culminating activity? Invite the mathematics teachers in your school or in other schools who are not members to join the Council and to make plans to be in Denver too at the end of August.

Important Issues in Arithmetic

By Ben A. Sueltz
State Teachers College, Cortland, New York

The issues formulated in this article have arisen frequently in conference with curriculum groups during the past five years. Repeatedly it has been found that tentative agreement upon basic ideas not only saves time but also points the way toward the formulation of major policies. The issues below are presented as questions with no conscious attempt to direct thinking toward any specific point of view or conclusion.

I. The Philosophy of Arithmetic Instruction

1. Can we accept the newer philosophy described by the words *meaning* and *understanding* and if so, are we prepared to organize and pursue learning so that meaning and understanding result? Is there genuine danger that, in many schools and with many teachers, real meaning and understanding will not be carried to the point where children become functionally competent in arithmetic? Is there danger that teaching for meaning and understanding may become patterned and stereotyped so that such values as discovery, appreciation, and resourcefulness may be lost?

2. Is the philosophy embodied in the words *tools for use* necessarily limited to a strong emphasis upon abstract computations and drill procedures? Does the newer concept of "functional competence" connote a better or broader vision of arithmetic than the older *tools for use*? Does the *tools for use* view of arithmetic tend to fit better into the formal than the informal pattern of education or is that merely a coincidence in many schools?

3. Can we believe that the way in which a thing is learned is frequently as important as the quantitative end products of learning? Are residual values of importance gained from such approaches to and avenues of learning as may be described as "experience education" and are these values not to be found in more traditional modes of learning? How important is the role of the teacher rather than the apparent framework or pattern of education?

4. How much arithmetic can children understand? How much should they attempt to understand? Have we evidence that the child who really understands the number system of arithmetic or the niceties of long division has gained something more valuable than the child who has gained proficiency only and is satisfied with a procedure because "it works"?

II. Aims of Arithmetic Instruction

1. Can we agree that the final main aim of arithmetic instruction is the development of a functional competence to sense, to understand, to be able to solve, and to feel a compulsion to solve the mathematical situations in one's social, economic and cultural surroundings? Should aims be limited to the uses of arithmetic that children encounter or should "life needs" beyond childhood experiences be considered as well?

2. Is it possible and desirable to formulate all the aims of elementary school arithmetic to include such elements as concepts, ideas, information, principles, understandings, and judgments as well as the usual computations and applications?

3. Admitting the non-uniformity of pupils in terms of such factors as ability, experience, and will to learn, what differentiation of goals should be set in terms of these differences? Is there a minimum set of goals that all pupils should attempt to reach because society expects reasonably intelligent mathematical behavior? If at least minimum goals are expected of all pupils, should supplementary goals be established for "better pupils" and ad-

ditional goals for "exceptional pupils"?

4. Are definite goals desirable for each school grade and if so, should they be merely suggestive to the teacher or should they be positively met? Have some schools failed their responsibility to the extent that groups of pupils are permanently "caught short" because teachers have taken a very casual attitude toward achievement in arithmetic as well as in other fields? Is it true that pupils tend to learn largely what is expected of them?

5. Assuming that arithmetic is the most useful part of mathematics for the general citizen and knowing that the elementary school cannot provide this adequately, what provision must be made for an arithmetic for "young adults"? What are the features of such an arithmetic and at what level will it be taught?

III. Modes of Learning Arithmetic

1. Is it generally true that learning based upon experience in seeing and handling real things and learning for an immediate personal need result in more facile learning and longer retention than more direct and systematized learning? Is the real issue in determing what things are more readily understood by one approach and what phases of arithmetic are better learned by another approach?

2. Must we face the probability that some pupils apparently learn more readily and perform better by one approach than by some other method and that this is not uniform from grade to grade or within a given group of pupils? If such is the case, should we provide for these differences? How?

3. What significance should be attached to the role of pupil discovery in learning arithmetic? Is it better for a pupil to be an active participant in discovering for himself the fact "$3 \times 6 = 18$" than merely to accept the conclusion? Does the mental level or the experience level of the pupil have a bearing upon this question?

4. To what extent should modes of learning arithmetic be conditioned by subsequent study of mathematics? For example, should pupils in lower grades learn a process such as subtraction by a procedure that will suffice for later work?

5. Are there certain concomitant values such as neatness, orderliness, and the feeling for surety and exactness that can be derived by attention to modes of learning arithmetic? Is there any evidence that these concomitant learnings, if achieved in the elementary school, persist over many years?

IV. School Practices in Arithmetic

1. Is it generally true that school practice remains largely the same as 20 years ago? Does this apply to point of view toward arithmetic and children, to modes of learning, and to expected outcomes? If this is true, how can our better teaching procedures be translated into more common practice?

2. Can arithmetic be learned satisfactorily without a separate period devoted to instruction therein? Are certain phases of arithmetic, as for example, computations, best learned in an organization that adheres to natural sequence and capitalizes upon mathematical relationships?

3. Drill in arithmetic continues as a problem. Does the main issue lie in knowing when to use drill and what kinds of drill are best suited for a specific purpose rather than in trying to determine how much drill is needed for each type of learning? Is drill or practice upon such elements as thinking, understanding, and judgment as necessary as drill upon computations?

4. What is the relation of arithmetic to the total school program? How much time should be devoted to materials that are primarily arithmetic at the several age-grade levels? What integrations can and should be made with other fields of study? How does the philosophy of child development affect our view of the content,

organization, and modes of learning arithmetic?

V. Evaluation and Research in Arithmetic

1. Is it possible to test and evaluate all of the important goals and aims in arithmetic? What media and modes of evaluation are best suited for different aspects of arithmetic? Can functional competence in the real world be evaluated in the school?

2. What is the valid educational role of standardized tests? How can guidance be given so that school people generally may evaluate testing materials in terms of content, significance, and valid interpretation?

3. Researches have tended to be "piecemeal" in that they have been necessarily limited in area and scope. Can the results of such researches be summed into a large or total pattern of education? Must the larger questions of arithmetic instruction continue to be settled on the basis of "informed judgment"? Is there a technique for arriving at "informed judgment"?

4. How can the many and widely scattered researches dealing with arithmetic be made available to interested groups? Is the real need a critical appraisal and a synthesizing of research results rather than a simple compilation?

5. Is current research throughly reliable? Or do such contingencies as method, point of view, and incentive limit the results to situations similar to those in the original research? What new types of experimental studies and researches are needed in terms of our changing view toward children and elementary education?

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◆ THE ART OF TEACHING ◆

Rectangular Coordinates and Signed Numbers in Seventh Grade Mathematics

By MAX A. SOBEL
State Teachers College, Montclair, New Jersey

IN DISCUSSING modern trends in mathematics, Professor Reeve of Columbia University, has often mentioned the gradual "leveling down" in this field. That is, instead of introducing various phases of the subject at an earlier stage of the teaching process, it is deferred until later. Thus we are now able to find college algebra texts that should provide no challenge to a good high-school group.

Since I do not believe this trend to be a favorable one, I am eager to report on a successful experiment whereby I introduced my 7th grade class to rectangular coordinates and signed numbers at least one year before the time when it is normally taught. By doing this, it is my belief that fewer will be left stranded when they finally meet these topics in detail in 9th grade algebra.

Our experiment began with a unit on scale drawings. After several days of class work we went outside in groups to measure the quadrangle in front of the College High, and then proceeded to make a scale drawing of this field. When finished, I gave the group a glimpse of the thrills of indirect measurement by asking them to find the distance between two points where a large tree and bench prevented direct measurement. To locate my two chosen points, I instructed all to find point A by going 80′ to the right and 45′ up from the lower left hand corner of the drawing. Similarly for point B.

After locating several such points and measuring distances, I suggested that it was difficult to say "go over x feet, and up y feet" each time, and asked whether I might just write (80, 45) instead. All agreed it was a fine timesaver. We mentioned that it was imperative to have a fixed starting point, and agreed to choose the lower left hand corner as our arbitrary origin.

We located many more points, all written in the form (x, y), and then left the project for several days. Upon our return to scale drawings the following week, the class made a map of a camp which I described to them. They easily constructed the swimming pool after I had given them, in coordinate form, the four corners. For further practice we located a tennis court, house, etc., in a similar manner, there being no abstract location of points.

Having completed this, I told the class that I was troubled: Our drawing was almost complete except for a baseball field which was not directly on the camp grounds. As a matter of fact, it was to the left of the entrance, the origin we had previously established. My problem was to locate the field for the students, and an immediate response was offered that I say "go left, then up."

We agreed on this solution, but I stated that it was too tiresome and clumsy to always write "go left, then up." One youngster then exclaimed that we could symbolize this by (82L, 40). Almost immediately someone else suggested, (before I could), that we write $(-82, 40)$ instead. A discussion ensued as to which one we should use, the class favoring the former symbol.

At this point I interjected a story stat-

ing that this had happened many years ago, at which time it was decided to use the minus symbol, a tradition mathematicians have followed ever since. This was enough to bring about the adoption of the conventional symbolism.

After this it was a simple task to get the class to decide that "right, down," and "left, down" could be represented by the appropriate combination of plus and minus symbols. Other concrete examples (the location of ships at sea, map directions, etc.) followed. We had thus introducted the concepts of rectangular coordinates in a genuine meaningful manner, and had also started the beginnings of signed numbers at an early stage of development.

Our work did not end here; it would not otherwise be worth the effort involved. A unit on graphing followed and it was now quite natural, for example, to graph temperatures which ranged above and below zero, involving again the use of signed numbers. We also graphed a chart of past and predicted temperatures so that $(-2, -5)$ had full meaning: two days ago it was 5 degrees below zero.

The alert teacher can readily enlarge on this program and thus pave the way for clearer, deeper, and certainly easier comprehension of these two important topics when finally taken up in full detail in later work.

Duke Mathematics Institute

The ninth annual session of the Institute for Mathematics Teachers in High School and College will be held at Duke University, Durham, N. C., August 8–19, 1949.

There will be ten study groups under the leadership of teachers of national recognition. The general theme of the Institute will be "Mathematics at Work."

Study Groups:
I. Aids in the Study of Algebra
II. Outdoor work in Mathematics
III. Modern Geometry with Applications
IV. Arithmetic of Home, Business, and Industry
V. Graphical Analysis with Applications
VI. Junior High School Mathematics
VII. The Slow Student in Mathematics
VIII. Applications of Mathematics (High School—College)
IX. Laboratory Mathematics (Junior-Senior High School)
X. Laboratory Mathematics (High School—College)

Twenty speakers of note will be on the program representing industry, business, education, science, and engineering. The complete program will appear in the May issue.

Special emphasis will be given to social events during the Institute.

Programs with detail information will be ready April 1.

W. W. Rankin, Professor of Mathematics, Duke University
Director of Mathematics Institute

AIDS TO TEACHING

By

HENRY W. SYER
School of Education
Boston University
Boston, Massachusetts

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.21—*Teaching Mathematics Through School Savings*

Education Section of the U. S. Savings Bonds Division, Treasury Department, Washington, D. C.
Booklet: 31 pages; Free; Irene M. Reid, Author.

Description: This booklet gives information, activities, and problems about school savings with emphasis on U. S. Savings stamps and bonds. It describes important reasons for investing in U. S. securities and presents statistics and graphs relating to Bond and Stamp sales. A list of mathematical concepts is presented with suggested activities and problems on such topics as budgeting, interest average, inflation, banking, ratio, per cent, rounded numbers, and graphing. Projects on saving and spending with emphasis on the social phase of mathematics instruction and data for making problems are presented.

Appraisal: This free booklet will furnish the teacher of grades 7–9 with much usable material for the mathematics class. Although almost all the material deals with U. S. Savings stamps and bonds, the projects, activities, and problems will suggest many ways of supplementing instruction with up-to-date materials and experiences.

CHARTS

C.7—*Trigonometric Functions*

Monroe Calculating Machine Company, P. O. Box 540, Orange, New Jersey
Chart; 11″×14″, Free.

Description: The thin white cardboard chart is folded to produce a four-page notebook-size table. The first page contains information concerning trigonometric functions of the right triangle. A line diagram visualizes the concepts of sine, cosine, tangent, etc. The Pythagorean theorem and that concerning the sum of the angles of any triangle appear below the diagram. The formulas for determining trigonometric functions are also given. Several of the formulas given are of little use, e.g., formulas for the vers, covers, exsec, and coexsec.

Pages two and three are devoted to solutions of the oblique triangle. Nineteen line diagrams accompany the nineteen formulas presented; known parts of the triangle are printed in heavy type. All but three of the formulas are derived from the Law of Sines; the remaining three treat the Law of Tangents.

The remaining page of the chart contains suggestions for using the Monroe Adding-calculator to solve trigonometric problems. This page is included for the convenience of those who must solve many such problems in engineering work.

The chart is neat, well arranged, and contains useful formulas but is too small to hang in the classroom for reference. It is useful as an individual aid rather than as a device for display.

Appraisal: The chart may be useful for students who have not yet become adept in manipulating formulas. It may be used as a check on their ability to determine the best form of a formula for a specific purpose. An advantage of the chart is its compactness of formulas and accompanying diagrams.

The last page is somewhat useless for secondary school purposes but may serve to illustrate that long methods of solution are usually discarded by those who have many calculations to perform. It might prove worthwhile to demonstrate how the adding-calculator reduces the time involved in solution of trigonometric problems. If the charts are obtained in quantity they may be advantageously distributed. If only one copy is desired, it could be hung or placed where it will be accessible to the students. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

EQUIPMENT

E.8—*Imma Whiz*

Kenworthy Educational Service, 45 North Division Street, Buffalo 3, New York.

One game for group drill in the basic addition and subtraction facts and one game for group drill in the basic multiplication and division facts. Price per game, $1.00.

Description: The addition and subtraction set has 24 6″×6″ cards, each of which has six addition and six subtraction combinations without answers. The multiplication and division set has 24 6″×6″ cards each of which has six multiplication and six division combinations without answers. The answer cards are separate from the combination cards, and as the teacher draws an answer card and reads it, pupils place markers on combinations which have this answer. The pupil getting a row or a column complete first is the winner. He must then read back the answers as well as the combinations to see that he does not have any errors.

Appraisal: The directions for these two games state that "the keen competition stimulates pupils to learn combinations quickly, accurately, and permanently." This claim is open to question. If these games are used while children are *learning* the combinations, the value may be negative. Under duress of competition, the answer becomes the only important consideration and immature processes such as counting and partial counting may become habituated. The use of the games should be deferred until children have completed the learning stages for the facts. When the children's thinking has been reorganized through a program of meaningful development to the point where they are really automatically responding to the combinations, the use of these games will provide one approach to the drill program.

The combinations and answers are printed on tagboard, but appear to be fairly durable. The printing is of good size and is clear. Because one set can serve up to twenty-four pupils, the cost is reasonable.

No specific recommendations can be be made as to the grade levels for these games because the developmental program for basic number facts varies from school to school. When children understand the facts and when they have actually reached the point where they are responding immediately to the presented combinations, they are ready for repetitive practice and these games will probably be helpful. (Reviewed by Robert L. Burch, Boston University, Boston, Massachusetts.)

FILMS

F.33—*Fred Meets a Bank*

Coronet Instructional Films, Coronet Building, Chicago 1, Illinois

Educational Collaborators: I. Owen Foster, Indiana University; and Frederick G. Neel, Canterbury College

16mm sound film, 1 reel; Black and white —$45, color—$90; 1947.

Content: Fred Marshall, 12 years old, considers what to do with a $10 birthday check received from his uncle and $8.50 cash earnings from his paper route. Fred's father explains checking accounts and savings accounts to him, and suggests that Fred use part of his money to open a savings account at the local bank. Next

morning Fred goes to the bank, an actual one, and is shown completing the several steps necessary to open a new account. He then meets his father, who is at the bank for the purpose of arranging a building loan. Fred witnesses this transaction, and then accompanies his father to the latter's safety deposit box in the bank vault. Before leaving the bank, Fred is also taken behind the scenes to learn something of the bookkeeping necessary to the bank's business.

Appraisal: This film does not attempt to teach specific mathematical processes. It is a good general survey of the ordinary banking services and procedures, presented in a manner which is understandable and interesting. The action and dialogue are easy and natural. The sense of forced artificiality which unfortunately characterizes some educational films is notpresent in this film. The fact that most of the action takes place in an actual bank gives a feeling of authenticity. This film would be very suitable for introducing units on checks and savings, or on almost any phase of general banking. It could be used in summarizing such units. It would be excellent for providing background for a field trip to a bank. The film develops a good illustration of one of the real life uses of some branches of school mathematics. A teachers' guide is included with the film.

Technical Qualities: Photography: very good. Commentary and dialogue: Distinct and appropriate. Content: Excellent. Grade level: 6–10. (Reviewed by Elwood M. Stoddard, Hingham, Massachusetts.)

F.34—*Indirect Measurement*

Knowledge Builders, 625 Madison Avenue, New York

Film; 1 reel, 16 mm; sound; black and white; $40; 1948.

Content: This film demonstrates three methods of indirect measurement-congruent triangles, similar triangles, and trigonometry. Congruent triangles are used to measure the width of a river; similar triangles are used to determine the height of a tree; trigonometry is used to solve a navigation problem in which the angle is measured by a sextant. The reading of a table of trigonometric functions is included in the navigation problem.

Appraisal: Indirect measurement is an appropriate topic for treatment by a film. However, a film should not replace field work with measuring instruments such as a transit. The film brings to the classroom illustrations of the use of indirect measurement in specific situations. Animated drawings are used to illustrate straightforward applications. The problems might be more realistic if the film had included photographs of real situations. The use of the film to show the reading of a table is effective. The students will need to know the basic concepts of indirect measurement if they are to understand the problems presented in the film. The photography and commentary are satisfactory.

F.35—*Pythagorean Theorem*

Knowledge Builders, 625 Madison Avenue, New York

Film; 1 reel, 16 mm; sound; black and white; $40; 1948.

Content: This film presents the historical background of the Pythagorean theorem in China and Egypt and includes a detailed discussion of an interesting use of the 3-4-5 triangle by the Egyptians to determine the east-west line. The theorem is proved and problems illustrating the use of the theorem are solved.

Appraisal: The background material and applications presented in this film will be welcomed by teachers of geometry and trigonometry classes. The proof of the theorem can usually be done as well by the teacher or textbook as by the film. In order for students to follow the problems solved, they will need to know how to use radicals. The photography and commentary are satisfactory.

FILM-STRIPS

FS.48—*Introduction to Signed Numbers*

Society for Visual Education, 100 East Ohio Street, Chicago, Illinois

Filmstrip; 35 mm; 45 frames; black and white; $2; 1948.

Content: The need for signed numbers to represent opposite values is illustrated by high and low temperature readings. A vertical scale and a horizontal scale with numbers on both sides of the zero point are used to explain the meaning of positive and negative numbers. The absolute value of a number is defined and illustrated. Examples of the uses of signed numbers include altitude readings, bank accounts, latitude and longitude readings, and historical dates. The filmstrip uses eight frames to review the material presented with six questions and answers.

FS.49—*Addition and Subtraction of Signed Numbers*

Society for Visual Education, 100 East Ohio Street, Chicago, Illinois

Filmstrip; 35 mm; black and white; $2; 1948.

Content: The addition of signed numbers is explained by comparing earnings (positive numbers) with expenditures (negative numbers) to determine the balance or savings. The subtraction of signed numbers is taught by showing a boy walking on a horizontal scale from one number to another. The distance walked and the direction walked are used to show the answer to subtraction problems. The rules for addition and subtraction are stated following the illustrations. Eight frames are used for asking questions.

FS.50—*Multiplication and Division of Signed Numbers*

Society for Visual Education, 100 East Ohio Street, Chicago, Illinois

Filmstrip; 35 mm; black and white; $2; 1948.

Content: Multiplication by a positive number is explained by considering multiplication as repeated addition. Multiplication by a negative number is explained by relating it to a non-mathematical situation where good citizens are considered a positive quantity and lawbreakers a negative quantity, rewards (+) and punishment (−). Similarly division of signed numbers is explained entirely in terms of non-mathematical conditions. The rules for mathematical computation are stated. Eight frames are used for stating eight questions or problems.

Appraisal of FS.48, FS.49, and FS.50: It is the opinion of the reviewer that the fundamental processes with signed numbers should be presented in such a fashion that the pupil is led to discover the method of computing on the basis of the meaning of the process and that the rules of computation should be introduced only after considerable contact with the process. This is not the method of these filmstrips. The statement of rules of computation or definitions are introduced too soon. On the other hand most of the content is too elementary to be used for a review. The appropriateness of using a non-mathematical situation to derive the rules for multiplication and division is questionable. Except for the pictures of applications, these filmstrips do little that cannot be done as well or better by blackboard illustration. Many frames are wasted asking question and then giving the answers to these questions.

INSTRUMENTS

I.7—*Pantograph*

J. L. Hammett Company, Kendall Square Cambridge 42, Massachusetts. $2.00

Description: The pantograph is composed of four fifteen-inch varnished, graduated wooden legs connected to form a collapsible parallelogram with extended legs. The extremity of one leg is fastened to the blackboard frame and remains fixed as the operator traces a picture with a

point on another leg; a third point reproduces a similar diagram upon the blackboard in a scale dependent upon the setting of the bars.

Appraisal: Linkages in general are not used to any great extent in geometry classes but the pantograph is often present. Use of the instrument as a drawing device depends upon similar triangles; the pantograph may thus be studied as an interesting application of geometry. Other less familiar linkages may be studied to determine their geometric principles.

A single pantograph is usually sufficient for the needs of the mathematics department. The price of this instrument is quite high but is not excessive unless the school has facilities for making it more cheaply. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

MODELS

M.5—*Planisphere*

J. L. Hammett Company, Kendall Square, Cambridge 42, Massachusetts.

10″ diameter, $3.

Description: The Planisphere is a circular cardboard disk upon whose surface is a good diagram of the principal stars visible in the northern latitudes. The disk is enclosed within a cardboard cover whose edges contain scales which indicate which stars are in the observable heavens at any given hour during the year. The instrument is manipulated by revolving the disk until the day of the month desired is opposite the correct hour of the day. Then, an opening in the cardboard enables the operator to see those stars visible in the heavens, whereas those obscured beneath the horizon are similarly obscured by mechanical means in the case of the aid.

The Planisphere is extremely well made of durable cardboard attractively finished in blue, yellow, and black. The stars are printed clearly in white against a black background.

Appraisal: In recent years astronomy has been omitted from many secondary curricula. Its study may be introduced in the solid geometry class if only to correlate the location of stars with coordinate systems employed to locate points in space. The Planisphere may be used for this purpose together with the "Observascope" which was reviewed in April, 1948 (See I.4)

The study of elementary astronomy in the mathematics classroom is relevant and suited to the abilities and backgrounds of secondary students. Such pertinent topics as the elliptical orbits of planets and changes in time and season due to planetary motion are already receiving attention by the mathematics department.

The Planisphere is not an essential piece of equipment by any means but it may be used as one means of making a phase of mathematics more interesting and realistic. It is used to best advantage during evening field trips. A larger model of this instrument is avilable for $7. It is superior in visibility and contains some additional information concerning motion of the planets. (Reviewed by Bernard Singer, Hyannis, Massachusetts.)

Algebra

Since I've studied algebra
There's one thing I've learned.
That is, the grades I got honestly
I surely have earned.

Alberta Watson, Grade 9
Cloverdale High School
Elbridge, Tennessee

◆ IN OTHER PERIODICALS ◆

By NATHAN LAZAR

Bureau of Reference, Research and Statistics
Board of Education, Brooklyn, New York

Conference on Arithmetic, 1948

1. Grossnickle, F. E., "Use of Multi-Sensory Aids in Developing Arithmetical Meanings," pp. 1–14.
2. Spitzer, H. F., "Procedures and Techniques for Evaluating the Outcome of Instruction in Arithmetic," pp. 15–25.
3. Storm, W. B., "Arithmetical Meanings that Should be Tested," pp. 26–31.
4. Cronbach, L. J., "Meanings of Problems," pp. 32–43.
5. Hartung, M. L., "Advances in the Teaching of Problem-Solving," pp. 44–53.
6. Schneider, H. A., "Place of Workbooks in the Teaching of Arithmetic," pp. 54–67.
7. Wingo, G. M., "Organization and Administration of the Arithmetic Program in the Elementary School," pp. 68–79.
8. Wren, F. L., "Professional Preparation of Teachers of Arithmetic," pp. 80–90.

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Conference on Teaching Mathematics, Grades 1-12

Sponsored by the Department of Mathematics and the School of Education, The University of Wisconsin

Theme: The learning of mathematics is a continuous growth from earliest number experiences to graduation

JULY 18–22

For additional information and complete program write Professor J. R. Mayor, North Hall, The University of Wisconsin, Madison

Where the program is printed in two columns, the column on the left gives the program planned for elementary school teachers and that on the right the program for secondary school teachers.

Those for whom no school is listed are members of the faculty of the University of Wisconsin.

JULY 18, MONDAY

8:00 A.M. Registration, Memorial Union Building

9:15 A.M. *Presiding:* PROFESSOR J. R. MAYOR

"Planning the Mathematics Curriculum as a Continuous Sequence," Professor F. L. Wren, Head of the Department of Mathematics, George Peabody College for Teachers, Nashville, Tennessee

1:15 P.M. *Presiding:* PROFESSOR C. C. MACDUFFEE

"Concepts Which Are Common to Mathematical Experiences at All Levels," Professor Harold Fawcett, Chairman of the Department of Education, The Ohio State University

7:00 P.M. Mathematics Films

9:00 P.M. Reception given by the Department of Mathematics

JULY 19, TUESDAY

9:30 A.M. *Presiding*: Professor V. E. Herrick

"Understanding Adds Interest," Professor W. B. Storm, Head of the Department of Mathematics, Northern Illinois State Teachers College, DeKalb

"Building Number Concepts," Professor Wren

Presiding: MISS THEDA FRANCES HOWE, Riverside High School, Milwaukee

"Important Emphases in Teaching Algebra," Professor Fawcett

1:15 P.M.

Panel: "Use of Multi-Sensory Aids in Building Concepts." Chairman, Mr. Harvey Cornell, Principal, Door-Kewaunee Rural Normal School, Algoma

Panel: "Guidance in Senior High School Mathematics." Chairman, Miss Margaret Joseph, Shorewood High School, Milwaukee

Presiding: DEAN PAUL L. TRUMP

"Important Emphases in Teaching Geometry," Professor Fawcett

7:00 P.M. Picnic at Picnic Point

Picnic at Picnic Point

JULY 20, WEDNESDAY

9:30 A.M. JOINT SESSION WITH THE INSTITUTE FOR PRINCIPALS AND SUPERINTENDENTS
"The Life Adjustment Program," Mr. Galen Jones, United States Office of Education

1:15 P.M. *Presiding:* PROFESSOR R. E. LANGER
"The Place of Mathematics in General Education," Professor Emeritus William David Reeve, Teachers College, Columbia University

5:00 P.M. Trip to points of scenic and historic interest.

JULY 21, THURSDAY

9:15 A.M. *Presiding:* PROFESSOR LOIS NEMEC

"What Topics Should be Considered in Upper Elementary Grades and Junior High School," Miss Lenore John, The Laboratory Schools, The University of Chicago

Panel: "Planning Activities to Emphasize Meanings," Chairman, Miss Lucy E. Smith, McKinley Junior High School, Racine

Panel: "Providing for Future Scientists and Engineers." Chairman, Mr. Clifford Hertenstein, Senior High School, Beloit

"College Training and Job Opportunities for the Mathematics-Science Major," Professors B. H. Colvin and E. S. Sokolnikoff

1:15 P.M.

Demonstration Class, Grades 5 and 6 taught by Miss John

Presiding: MISS MARGARET SPIELMACHER, Central High School, Superior

"Enrichment Courses for Gifted Pupils in Mathematics and Science," Professor Reeve

7:00 P.M.

Dinner, Tripp Commons

Dinner, Tripp Commons

JULY 22, FRIDAY

9:15 A.M. *Presiding:* MISS MAYBELL BUSH, Wisconsin State Department of Public Instruction

Panel: "The Social Phase of Arithmetic." Chairman, Mr. H. Ralph Allen, Principal, Randall School, Madison

"The Need for Arithmetic in Retailing," Miss Mabel Carlson, Training Supervisor, Ed. Schuster and Co., Milwaukee

Presiding: MR. B. L. BOONE, South Division High School, Milwaukee

Address by representative of International Business Machines Corporation

Panel: "Community Applications of Mathematics." Chairman, Miss Margaret Striegl, Wauwatosa High School

1:15 P.M. Visit to the University Computing Laboratory
Supervised by Professor K. J. Arnold

STUDY GROUPS

Each study group will meet once a day for the five days of the Conference. Study Group periods are as follows:

Groups I, II, III	8:00–9:00 A.M.
	(because of registration these Groups will meet at 10:45 A.M. rather than 8:00 A.M. on Monday)
Group IV	2:45–4:45 P.M.
Groups V, VI	2:45–3:45 P.M.
Groups VII, VIII	4:00–5:00 P.M.

I. "The Mathematical Content of the Secondary School"
Leader: Professor Reeve

II. "Problem Solving in Mathematics at the Elementary School Level"
Leader: Mr. Paul W. Eberman, Department of Education, The University of Chicago

III. "Junior High School Mathematics"
Leader: Miss Edith Woolsey, Sanford Junior High School, Minneapolis, Minn.

IV. "Laboratory in Mathematics" (Construction and Use of Models and Instruments)
Leader: Miss Mary A. Potter, Mathematics Supervisor, Racine Public Schools

V. "What Are Meanings in Mathematics?"
Leader: Professor Wren

VI. "Understandings that Should Be Developed in Arithmetic"
Leader: Professor Storm

VII. "Audio-Visual Materials and Methods in Mathematics"
Leaders: Mr. Charles Schuller and Mr. Harold Hailer

VIII. "Surveying and Use of Instruments"
Leaders: Professor E. C. Wagner, Civil Engineering

The rapid rate at which mathematics is growing makes it almost impossible for teachers to keep from becoming antiquated. Most of us have neither time nor energy to study courses beyond the fields in which we are teaching.

The Mathematics Magazine is providing a means for obviating this difficulty easily and quickly. It is publishing a series of chapters on **"The Meaning of Various Courses in Mathematics"** or, as some call them, **"Understandable Chapters in Mathematics."**

Sixteen carefully selected authors are writing twenty-two chapters beginning with elementary algebra and covering most of the classic courses.

Starting in the March-April issue 1948, articles have appeared on: "Fundamentals of Beginning Algebra;" "The Meaning of Plane Geometry;" "The Grammar of Algebra;" "The Basic Concepts of Trigonometry;" "Analytic Geometry, the Framework of Mathematics," and "The Gist of the Calculus." An article on Topology by Professor D. W. Hall will appear in the May-June issue, followed by one on "Abstract Spaces" by Prof. Maurice Frechet, one on "The Elementary Theory of Numbers," by Prof. E. T. Bell, and others to be announced later.

Other major features of the Mathematics Magazine are: expository and research articles in the advanced and collegiate fields, problems and questions, discussions of published papers and mathematical items of especial interest.

The Mathematics Magazine is published by-monthly except July-August. The subscription price is $3.00; the present volume plus the March-April and May-June issues of 1948 can be had for $4.20.

Address:

MATHEMATICS MAGAZINE
405 Hilgard Avenue, Los Angeles 24, California

EDITORIALS

Precedence of Signs in Algebra

A LETTER to the editor recently contained a question from a classroom teacher of mathematics that has bothered many teachers of algebra for a long time. He referred to the fact that in one well known algebra the rule for doing a series of ordinary fundamental operations is stated as follows: "In a special series of operations involving addition, subtraction, multiplication, and division of numbers, the multiplications and divisions are performed in the order in which they occur, the additions and subtractions in the resulting expression are then performed in any order." In another equally prominent and widely used algebra the authors state the rule as follows: "To evaluate a complicated expression involving the four fundamental operations

1. Do all the multiplications.
2. Do all the divisions, taking them in order from left to right.
3. Then do the additions and subtractions taking them in any order."

Our correspondent further says that these rules "do not always give the same results when applied to a series of fundamental operations" and he wants to know why it would not be possible to get a uniform rule for such operations in algebra "whereby all teachers of algebra would teach the same rule and get the same results." This seems to be a sensible and fair request. Perhaps the National Council of Teachers of Mathematics might take up this question at some future meeting.—W.D.R.

The Awful Effect of Formalism and "Deadwood" in Algebra

Recent examination of some of the newer ninth grade textbooks in algebra reveals an alarming amount of formalism and "deadwood" in spite of all the reform and changes in content and emphasis upon certain topics by such well known reports as that of the recent Joint Commission and the Commission on Post-War Plans. A great many teachers take such reports seriously and practically all of the authors of textbooks pay lip service in their prefaces to such reports and then proceed to include in their books two types ofobjectionable material.

1. Material that is well known as obsolete like most types of factoring and complicated polynomial operations.

2. Material that is all right *per se* but which is now generally considered too hard for most ninth graders and which will surely be included in a better and more complete way in intermediate algebra. A good example of such material is the solution of quadratic equations by the method of completing the square or/and the formula. It is no longer required by any governing body.

What is the use for the Council and other similar groups to go to all the trouble and expense to try to help classroom teachers improve their teaching when some of the very people who serve on such committees, but who are incidentally authors of textbooks, do not follow their own real beliefs when they sit down to write an algebra? Some authors try to say that this formalism and deadwood is what a great many teachers want, but it does not square with the facts. Most of our teachers are not as antiquated as that. Authors of textbooks should lead—not follow. What is to be done about it?—W.D.R.

NEWS NOTES

Dr. H. Gray Funkhouser of Phillips Exeter Academy spoke at the Mathematics section of the New Hampshire State Annual Convention at Concord on October 20th. His subject was "Development of American Textbooks in Mathematics." Dr. Funkhouser's collection of early American textbooks were on display at the meeting.

At the same meeting Mr. Leslie S. Cummings gave a preliminary report for the committees on Reorganization of the State Program in Mathematics.

Mr. Merle Goodrich presided at the meeting.

The present officers of the Mathematics sections of the Maryland State Teachers Association were recently reelected for 1948–49. They are:

Chairman—Miss Margaret Bowers, Elkton High School, Elkton, Md.

Co-Chairman—Miss Margaret Heinzerling, Southern High School, Baltimore, Md.

Secretary—Miss Margaret Hamilton, Ft. Hill High School, Cumberland, Md.

Treasurers—Mr. Alfred E. Cully, Forest Park High School, Baltimore, Md., and Mr. J. Lerda, Westminster, Md.

The First Meeting of the Men's Mathematics Club of Chicago and Metropolitan Area was held on October 15, 1948, at the Central Y.M.C.A.

Professor W. T. Reid, of Northwestern University, spoke on "Soap Films and the Calculus of Variations. Mr. Glenn Anderberg was program chairman.

PROGRAM COMMITTEE

Glenn Anderberg, *Chairman*, Waukegan Township High School.

William C. Krathwohl, Illinois Institute of Technology.

Joseph A. Nyberg, Hyde Park High School.

OFFICERS 1948–1949

W. G. Hendershot, *President*, Roosevelt High School, Chicago, Illinois.

David Rappaport, *Secretary-Treasurer*, Lane Technical High School, Chicago, Illinois.

H. T. Davis, *Honorary President*, Northwestern University, Evanston, Illinois.

E. L. Tierney, *Recording Secretary*, Tilden High School, Chicago, Illinois.

SPECIAL PROJECT COMMITTEE

George F. Bradfield, *Chairman*, Lake View High School.

E. L. Paine, Downers Grove High School.

S. F. Bibb, Illinois Institute of Technology.

C. M. Austin, *Historian*, Oak Park, Illinois.

The John C. Winston Company announces the appointment of Dr. Charles A. Ford of Philadelphia as Editor-in-Chief of the Educational Department. Dr. Ford will succeed Dr. Glenn L. Lembke, who is returning to educational administration as Dean of Education at State Teachers College, Frostburg, Maryland.

The Mathematics Section of the Eastern Division of the Colorado Education Association held its Fall Meeting at Denver on October 28, 1948.

Theme—To unify the program of mathematics from the Elementary School through College.

Speakers—Mr. John Gates, Gates Rubber Company; Miss Bertha Stephens, Mr. Burnett Severson, and Mr. Donald McIntosh, of the Denver Public Schools.

MEMBERS OF THE EXECUTIVE COUNCIL

President, Prof. A. W. Recht, 2233 So. St. Paul St., Denver. *Vice-President*, Prof. Aubrey Kempner, Univ. of Colo., Boulder. *Secretary-Treasurer*, Burnett Severson, 3540 Belcaro Lane, Denver.

Term Expired 1948

Russell Casement, Denver; Sarah B. Duboff, Denver; Leota Hayward, Fort Collins; Ruth Hoffman, Denver.

Term Expires 1949

Ethel Gordon, Denver; Ivan Hebel, Golden; Margaret Murray, Denver; Dorothea Williams, Denver.

Term Expires 1950

Ruth Bigham, Denver; Helen E. Howarth, Denver; Kenneth Noble, Denver; Opal B. Riddell, Denver.

The section publishes an official bulletin. Miss Ruth Irene Hoffman of Denver is editor.

A HIGH SCHOOL MATHEMATICS CONFERENCE WAS HELD AT MILLERSVILLE STATE TEACHERS COLLEGE, MILLERSVILLE PA. ON OCTOBER 2, 1948.

PROGRAM

9:00 A.M.—Mathematics Exhibit, Dining Room Foyer.

9:30 A.M.—Morning Session, Chapel Auditorium.

Call to Order, Dr. Lee E. Boyer, *Chairman*, Mathematics Department.

Address of Welcome, President D. L. Biemesderfer.

Address: A New Era in Calculating Machines, Dr. H. D. Huskey, *U. S. Bureau of Standards, Washington, D. C.*

Discussion: Leader, Mr. George R. Anderson.

Projection, Motion Picture Films.

Panel Discussion: Techniques in Teaching Mathematics, Norman B. Bucher, Lebanon, *Chairman.*

Harold Batorf, Myerstown; William Bolger, Palmyra; C. R. Eshleman, Lancaster;

Henrietta Flickinger, Hanover; George N. Leib, Dover; Hale C. Pickett, West Chester; Robert Shoff, Riverview (N. J.).
Dinner Meeting, College Dining Room.
Address: Wanted: a Two-Track Program in Mathematics, Dr. William Betz, *Specialist in Mathematics, Rochester, N. Y.*
Discussion, *Leader*, Dr. Frederick L. Pond, *State Department of Public Instruction, Harrisburg, Pa.*

The Mathematics Section of the Minnesota Education Association held its Fall Meeting on Thursday, October 28th, 1948 with Donovan Johnson, of the University High School presiding. The meeting was held in Minneapolis. The program follows:

Theme: "New Trends in Teaching Mathematics," Chairman: Theodore Kellogg, University of Minnesota High School.
"Concrete Expression in Mathematics," Emil J. Berger, Monroe High School, St. Paul.
"Cartoons for Teaching Mathematics," Marvin L. Johnson, Junior High School, Rochester.
"Mathematics and Computing Machines," LeRoy Simon, College of Education, University of Minn.
"Films and Filmstrips for Geometry," Donovan A. Johnson, University of Minnesota High School.
"Workshops for Mathematics Teachers," Clarice Kaasa, Senior High School, Red Wing; Edith Woolsey, Sanford Junior High School, Minneapolis.
Business Meeting.

Mrs. Oliver Shurtleff of the mathematics department of Morris Harvey College, Charleston, and Dr. Joseph Stewart of the mathematics department of West Virginia University, Morgantown, were speakers at the Mathematics luncheon held during the eighty-fourth annual convention of the West Virginia State Education Association, November 11, 1948, Charleston, West Virginia. James Kessel of Ripley presided.

Kathryn W. Lynch, Saint Albans, was elected Chairman of the Mathematics Section of the SEA for next year. Mary Jo Blackburn, Charleston, was elected Secretary.

The December meeting of the Women's Mathematics Club of Chicago and vicinity was held on Saturday, December 4, 1948, at 11:45 A.M. in Mandel's Tea Room.

The meeting was a panel discussion devoted to the topic "Creating a Rich Course in Trigonometry for High School Students." The chairman of the panel was Miss Mildred Taylor of Fenger High School. Taking part in the panel were Mr. Ralph Briggs of Bloom Township High School, Mr. Glen Hewitt of Von Steuben and Mr. Willard Johnston of Fenger. The program was planned to give a wealth of practical ideas that can be used in the class room.

The Men's Mathematics Club of Chicago and the Metropolitan Area held its second meeting of the year 1948–49 on Friday, Nov. 19, at 6:30 p.m., at the Central Y.M.C.A.

Program

Topic—"The Bird's Eye View Versus the Worm's Eye View in Teaching Mathematics."
Speaker—Professor M. L. Hartung, Department of Mathematics and Education, University of Chicago.

The Fall Meeting of the Oregon Council of Teachers of Mathematics was held on Dec. 4th, 1948, at Salem. The program follows:

General Theme: Challenge to Teachers of Mathematics.
Responsibility in School Curriculum—Dr. Frank Bennett.
Opportunities in Teacher Training—Dr. George D. Martin.
Possibilities for Enjoyment in Mathematics—Dr. Chester F. Luther.

The 1948 Convention Program of the Central Association of Science and mathematics teachers was held at the Claypool Hotel, Indianapolis, November 26 and 27, 1948.

General Program

9:30 A.M. Friday.
"Recent Studies of the Atlantic Ocean Basins," Dr. Maurice Ewing, Columbia University; Leader of the Expedition to the famous Mid-Atlantic Ridge.
10:15 A.M. Friday
"Problems of Science and Mathematics in General Education," *Third Annual Forum:* AAAS Cooperative Committee on the Teaching of Science and Mathematics, Participating: Dr. K. Lark-Horovitz, chairman, Purdue University; G. P. Cahoon, Ohio State; Prevo L. Whitaker, Indiana University, Raleigh Schorling, University of Michigan.
11:15 A.M. Friday
General Mathematics for Grades Nine to Twelve," Dr. W. D. Reeve, Professor and Head of the Mathematics Department, Teachers College, Columbia University.
6:30 P.M. Friday, Annual Banquet
"How Successful Are We in Pur Efforts to Reform German Education," Dr. Leo J. Brueckner, Chief of Elementary Education OMGUS, Berlin, Germany; Professor of Elementary Education, University of Minnesota; Author
9:15 A.M. Saturday
"Our Navy Explores Antarctica," Captain of the U. S. Navy, George J. Dufek; Commander of Task Force 68, with Admiral Byrd during the 1946–47 Antarctic Expedition.

Mathematics Section

Philip Peak, University High School, Bloomington, Ind. presiding.
Modern Trends in Secondary Mathematics, Dr. W. D. Reeve, Professor and Head of the Mathematics Department, Teachers College, Columbia University.
Panel: Current Experiments in Mathematics Teaching.
1. Functional Geometry
H. Vernon Price, Iowa State University, Iowa City, Ia.
2. An Attempt to Measure Critical Judgment, Dr. Bjarne R. Ullsvik, Illinois State Normal, Normal, Ill.
3. Report of the Trends Committee.
Chairman: Philip S. Jones University of Michigan, Ann Arbor, Mich.

Mrs. Charlotte Grant of Oak Park, Ill., was elected president of the Central Association of Science and Mathematics teachers. She succeeds Dr. J. E. Potzger of Butler University.

Dr. Paul Trump of the University of Wisconsin was elected Vice President to succeed Miss Mary Potter of Racine, Wis.

At the meeting of the Oregon Council Teachers of Mathematics on December 4, 1948, the following program was given:

The Challenge to Teachers of Mathematics through Responsibility in School Curriculum," Dr. Frank B. Bennett.

"Opportunities in Teacher Training," Dr. George D. Martin.

"Possibilities for Enjoyment in Mathematics," Dr. Chester F. Luther.

Announcement of Special Summer Program at N. C. State College

The School of Engineering (with an engineering enrollment of 2500) of North Carolina State College has the facilities and staff sufficient to make available a unique opportunity for high school teachers of mathematics and physics. The School of Engineering proposes to offer this next summer a six-week program of three related and integrated courses to carry a total of nine quarter hours of credit and this credit will be accepted by the North Carolina State Board of Education for the renewal of "A"-grade certificates. The three courses can be described briefly as follows:

1. One class will meet for five one-hour periods and one two-hour drill period each week in mathematics. The material for this course will emphasize graphs and will range from topics in college algebra to a graphical explanation of calculus. Several topics will be included which are not customarily a part of freshman and sophomore mathematics but which will enrich the content of these courses (scales, the slide rule, contour plotting of surfaces, etc.).

2. A class meeting for five one-hour periods and one two-hour laboratory each week in physics will not be the traditional course in college physics but will be concerned more with tracing fundamentals through related phases of physics. Classroom demonstrations will be given of numerous physical phenomena.

3. An introduction to *what is engineering* will be given in a third class which will meet for five one-one hour periods and two inspection trips or laboratories each week. All of the engineering departments will cooperate in giving this course and will in turn describe the work their graduates do, the kind of mathematics their students need, and so on. Some of the laboratory periods will be devoted to actual engineering experiments where the teacher-student will be enabled to plot the data on a graph sheet and see the engineering law develop. Other periods will be in the nature of inspection trips to see, for example, how nylon is made or the difference between amplitude and frequency modulated radio waves.

The mathematics teachers taking this program will be encouraged to keep a notebook of application problems which, it is hoped, this technical environment will suggest. It is the hope of the teachers in this college that these high school teachers will be helped in their teaching of all students and not merely those going on to a study of some phase of technology.

Further information may be obtained by writing to Mr. E. W. Ruggles at N. C. State College, Raleigh, North Carolina.

Mystery

"Can this be compared
To an endless chain
Linked slowly together
As its knowledge we gain?

Or is it like a faithful tree
Mothered by time,
With stair-step branches
That all might climb?"

This truthful guess
Makes the mystery complete
Except for its foremost goal
That for you I'll repeat.

We who have worked
Thoughtlessly day after day
Suddenly came to the realization
That *to think* can pay.

Thus the mystery is solved
As you can readily see,
The missing subject
Was our Plane Geometry.

Alice Graves, Grade 10
Bloomington High School
Bloomington, Indiana

NEW BOOKS

Bernstein, S., *A Centenary of Marxism.* Science and Society, New York, 1948. 196 pp., $2.50.

Butler, Charles H., and Wren, F. Lynwood, *Trigonometry, Secondary Schools, Plane and Spherical.* D. C. Heath & Co., Boston, 1948. 360 pp. $2.60.

Churchill, Ruel V., *Introduction to Complex Variables and Applications.* McGraw-Hill Book Co., New York, 1948. 216 pp. $3.50.

Daus, Paul H., and Whyburn, William M., *First Year College Mathematics with Applications.* Macmillan Co., New York, 1949. 495 pp. $5.00.

Hart, W. L., *Intermediate Algebra for Colleges.* D. C. Heath & Co., New York, 1948. 316 pp. $2.50.

Hawkins, George E., and Tate, Gladys, *Your Mathematics.* Scott, Foresman & Co., Chicago, 1948. 592 pp.

Herberg, Theodore, and Orleans, Joseph B., *A New Geometry for Secondary Schools.* D. C. Heath & Co., Boston, 1948. 406 pp. $1.92.

Hilborn, C. E., *Mathematics for Use in Business.* Houghton Mifflin Co., Boston, 1948. 472 pp. $3.50.

Hille, Einar, *Functional Analysis and Semi-Groups.* American Mathematical Society, New York, 1948. 530 pp. $7.50.

Hodge, W. V. D., and Pedoe, D., *Methods of Algebraic Geometry, Volume I.* Cambridge University Press, London, 1947. 440 pp. $6.50.

Hooper, A., and Griswold, A. L., *A Modern Course in Trigonometry.* Henry Holt & Co., New York, 1948. 101 pp. $2.52.

Keller, M. Wiles, and Zant, James H., *Basic Mathematics, A Workbook.* Houghton Mifflin Co., Boston, 1948. 253 pp. $1.50.

Keyser, Cassius Jackson, *Mathematics as a Culture Clue (And Other Essays).* Scripta Mathematica, New York, 1947. 277 pp.

Kidd, Kenneth Paul, *Objectives of Mathematical Training in the Public Junior College.* McQuiddy Printing Co., Nashville, 1948. 167 pp.

Larson, H. D., *Rinehart Mathematical Tables.* Rinehart & Co., New York, 1948. 264 pp. $1.50.

Lennes, N. J., and Maucker, J. W., *A First Course in Algebra, Second Revision.* Macmillan Co., New York, 1949. 561 pp. $2.40.

Linker, J. B., Hill, M. A., *Mathematics of Finance.* Henry Holt & Co., New York, 1948. 83 pp.

Love, C. E., *Analytic Geometry, Fourth Edition.* Macmillan Co., New York, 1948. 306 pp.

McCoy, N. H., *Rings and Ideals.* Mathematical Association of America, Waverly Press, Baltimore, 1948. 216 pp.

McLachlan, N. W., *Modern Operational Calculus.* Macmillan & Co., Ltd., London, 1948. 218 pp. $5.00.

Mallory, Virgil S., Cooke, Dennis H., and Brownfield, Mary L., *Practice in Using Arithmetic.* Workbook 1, 80 pp. Workbook 2, 96 pp. Benjamin H. Sanborn & Co., Chicago, 1948.

Murnaghan, Francis D., *Introduction to Applied Mathematics.* John Wiley & Sons, New York, 1948. 389 pp. $5.00.

Radó, Tibor, *Length and Area.* American Mathematical Society, New York, 1948. 572 pp.

Rees, P. K., and Mouson, E. D., *Analytic Geometry.* Dryden Press, New York, 1948. 305 pp. $2.75.

Schaaf, William L., Mathematics, *Our Great Heritage. Essays on the Nature and Cultural Significance of Mathematics.* Harper & Bros., New York, 1948. 291 pp. $3.50.

Sigley, D. T., and Stratton, W. T., *Plane Geometry.* Dryden Press, New York, 1948. 242 pp. $2.25.

Simmons, H. A., *College Algebra.* Macmillan Co., New York, 1948. 619 pp.

Snader, Daniel W., *Algebra, Meaning and Mastery, Book One.* John C. Winston Co., Philadelphia, 1949. 502 pp. $1.65.

Trefftzs, Kenneth Lewis, and Hills, E. Justin, *Mathematics of Business and Accounting.* Harper Bros., New York, 1947. 317 pp. $3.00.

Underwood, R. S., and Sparks, Fred W., *Analytic Geometry.* Houghton Mifflin Co., Boston, 1948. 225 pp. $2.75.

Walling, S. A., Hill, J. C., *Modern Mathematics.* Cambridge University Press, England, 1948. 153 pp. $1.00.

Wolfe, John H., and Phelps, Everett R., *Practical Shop Mathematics, Third Edition.* McGraw-Hill Book Co., New York, 1948. 371 pp. $2.40.

Woods, Roscoe, *Analytic Geometry, Revised Edition.* Macmillan Co., New York, 1948. 306 pp.

WELCHONS-KRICKENBERGER: Plane Geometry

Revised Edition. This popular book emphasizes practical problems using geometry in engineering, aviation, architecture, and science. Generous provision is made for individual differences by means of grading exercises, starring theorems, and organizing the book flexibly. Exercise material is abundant and varied. Notable for its careful definitions, clear statements of theorems, illustrations of new theorems. *New 1949 copyright.*

SMITH-FAGAN: Mathematics Review Exercises

New Edition. Ideal for review, preparation for examinations, and as a supplementary text. The pre-tested problems cover all the topics required by the Comprehensive Mathematics Test of the College Entrance Examination Board. They include a *complete* review of arithmetic; elementary, intermediate and advanced algebra; plane and solid geometry; trigonometry. This New Edition contains additional material and two sets of multiple-choice questions like the present C.E.E.B. examinations, with over 100 questions in each set.

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